# Lab Worksheet (Week-7)

## Row-Reduced Echelon Form

1. Create a code in Python that can be called as a function (name it $r\_r\_e\_f$) that gives the row reduced echelon form of any given matrix, its rank, and the pivotal columns.
   Find the rref of $A$ and $B$ using the above mentioned function:

$$A = \begin{bmatrix} 1 & 3 & 4 \\ 5 & -9 & -8 \\ 4 & 7 & 8 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 6 & 7 \\ -1 & -5 & 1 \\ 1 & 9 & 0 \end{bmatrix}$$

**Algorithm**

// Input: $m \times n$ matrix A

// Output:$m \times n$ matrix in reduced row echelon form, rank and the pivotal columns

1. Create a function that take the matrix $A$ as input
2. Set i = 0, j = 0, rank = 0
3. While $i \leq m-1$ and $j \leq n-1$ ($m$=number of rows and $n$=number of columns)
4. Find position of maximum absolute value(Pivot element and position) from $A(i,j)$, $i \geq j$.
5. If pivot is equal to zero(or less than some tolerance value)
   (a) Set j = j + 1
   else
   (b) Perform $R_i \longleftrightarrow R_{pivot}$
   (c) Divide each element of row $i$ by $a_{ij}$, thus making the pivot $a_{ij}$ equal to one
   (d) For each row $x$ from 1 to $m$, with $k \neq i$ subtract row $i$ multiplied by $a_{xj}$ from row $x$.
   (e) Set $i = i+1$, $j = j+1$, $rank = rank + 1$;
   end if
   end while
6. Return transformed matrix A and rank.

2. Find the basis of the Column space of the below matrices.

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 1 & 0 & 1 \end{bmatrix} \text{ and } C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$