Endsem Rubric

Answer 1(i):(5 marks)

Hours of air-condition usage per day(x_i)	Monthly electricity bill in INR(thousand)(y_i^*)
2	4.5
3	5.0
5	7.5
7	10.0
9	13.0

Let equation of line will be

$$y^* = bx + a \qquad (*)$$

Take $f(x) = x$

$$\begin{bmatrix} \sum_{i=1}^{5} 1 & \sum_{i=1}^{5} x_i \\ \sum_{i=1}^{5} x_i & \sum_{i=1}^{5} x_i^2 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{5} y_i \\ \sum_{i=1}^{5} y_i x_i \end{bmatrix} \qquad (1)$$

$$\sum_{i=1}^{5} x_i = 2 + 3 + 5 + 7 + 9 = 26$$

$$\sum_{i=1}^{5} x_i^2 = 4 + 9 + 25 + 49 + 81 = 168$$

$$\sum_{i=1}^{5} y_i = 4.5 + 5 + 7.5 + 10 + 13 = 40$$

$$\sum_{i=1}^{5} y_i x_i = 4.5 \times 2 + 5 \times 3 + 7.5 \times 5 + 10 \times 7 + 13 \times 9 = 248.5$$

substitute these values into equation 1 we get

$$\begin{bmatrix} 5 & 26 \\ 26 & 168 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 40 \\ 248.5 \end{bmatrix} \tag{2}$$

Let

$$AX = B$$

where $A = \begin{bmatrix} 5 & 26 \\ 26 & 168 \end{bmatrix}$, $X = \begin{bmatrix} a \\ b \end{bmatrix}$ and $B = \begin{bmatrix} 40 \\ 248.5 \end{bmatrix}$

since $|A| = 164 \neq 0$

$\Longrightarrow$

$$X = A^{-1}B$$

$$A^{-1} = \frac{1}{164} \begin{bmatrix} 168 & -26 \\ -26 & 5 \end{bmatrix} = \begin{bmatrix} 1.0244 & -0.1585 \\ -0.1585 & 0.0305 \end{bmatrix}$$

So,

$$X = \begin{bmatrix} 1.0244 & -0.1585 \\ -0.1585 & 0.0305 \end{bmatrix} \begin{bmatrix} 40 \\ 248.5 \end{bmatrix} \tag{1}$$

$$X = \begin{bmatrix} 1.5793 \\ 1.2348 \end{bmatrix} \tag{2}$$

$\Longrightarrow$

$$a = 1.5793, b = 1.2348$$

put these values in equation $*$

$$y^* = 1.2348x + 1.5793$$
$$y = 1234.8x + 1579.3$$

(ii).
for $x = 8$

$$y = 1234.8 \times 8 + 1579.3 = 11457.7$$

Answer 2:(5 marks)
(i)

$$\mathbb{P} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1/2 & 1/2 \\ 1/2 & 0 & 1/2 \end{bmatrix}$$

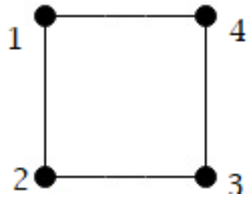
(ii)

Let $\vec{\pi}^{(\infty)} = \begin{bmatrix} \pi_1 \\ \pi_2 \\ \pi_1 \end{bmatrix}$ be the stationary (Invariant) distribution ,then

$$
\begin{aligned}
(\vec{\pi}^{(\infty)})^T &= (\vec{\pi}^{(\infty)})^T \mathbb{P} \\
\begin{bmatrix} \pi_1 & \pi_2 & \pi_3 \end{bmatrix} &= \begin{bmatrix} \pi_1 & \pi_2 & \pi_3 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1/2 & 1/2 \\ 1/2 & 0 & 1/2 \end{bmatrix} \\
\begin{bmatrix} \pi_1 & \pi_2 & \pi_3 \end{bmatrix} &= \begin{bmatrix} \frac{\pi_3}{2} & \pi_1 + \frac{\pi_2}{2} & \frac{\pi_2}{2} + \frac{\pi_3}{2} \end{bmatrix} \\
\implies 2\pi_1 - \pi_3 &= 0 \\
2\pi_1 - \pi_2 &= 0 \\
\pi_2 - \pi_3 &= 0 \\
\text{with } \pi_1 + \pi_2 + \pi_3 &= 1
\end{aligned}
$$

Solving this system we get $\pi_1 = \frac{1}{5}$,$\pi_2 = \frac{2}{5}$ and $\pi_3 = \frac{2}{5}$

Answer 3:(5 marks)



$$h_1(3) = ?$$

since

$$h_k(A) = 1 + \sum_{m \in S} P_{km} h_m(A) \qquad \forall k \in S \setminus A$$

so,

$$
\begin{aligned}
h_1(3) &= 1 + P_{13}h_3(3) + P_{12}h_2(3) + P_{14}h_4(3) \\
2h_1(3) - h_2(3) - h_4(3) &= 2 \qquad (1)
\end{aligned}
$$

Similarly we get below two equations,

$$h_1(3) - 2h_2(3) = -2 \qquad (2)$$

$$h_1(3) - 2h_4(3) = -2 \qquad (3)$$

from equation(2) and equation(3) we get,

$$h_2(3) = h_4(3) \qquad (4)$$

putting equation(4) in equation(1) we get ,

$$h_1(3) - h_2(3) = 1 \tag{5}$$

so,

$$h_1(3) = 1 + h_2(3)$$

Now putting equation (5) in equation(2) we get

$$h_1(3) = 4$$

Answer 4:(5 marks)

Solution: Consider that the requisite fps rating of the frontal camera system is n. Then, if $\bar{X}$ is the mean measurement made every second, then it is reasonable to assume that $\bar{X} \sim N\left(d, \frac{4}{n}\right)$ based on the CLT . Therefore,

$$\begin{aligned} P(-0.5 < \bar{X} - d < 0.5) &= P\left(\frac{-0.5}{2/\sqrt{n}} < \frac{\bar{X} - d}{2/\sqrt{n}} < \frac{0.5}{2/\sqrt{n}}\right) \\ &\approx P(-\sqrt{n}/4 < Z < \sqrt{n}/4) \qquad \text{(direct consequence of CLT)} \\ &= 2P(0 < Z < \sqrt{n}/4), \end{aligned}$$

where Z is the standard normal random variable. This means we must have $2P(0 < Z < \sqrt{n}/4) \geq 0.95$ or equivalently $\mathrm{P}(0 < Z < \sqrt{n}/4) \geq 0.475$. Following the standard normal distribution table , since we have $P(0 < Z < 1.96) = 0.475$; n must be chosen such that $\sqrt{n}/4 \geq 1.96$ or $n \geq 61.46$. This means that the fps rating of the frontal camera system of the autonomous car must be at least 62 .