

Instructions: You must **not** be in possession of any cheat sheet, notes, or electronic devices like laptops or calculators inside the examination hall. **Please answer all four questions.** Please begin your answer to a given question on a new page. Please **show all steps** leading to your final answer to receive any credit for your solution. Merely stating the final answer may not fetch you any credit. Maximum point allotted to each question is mentioned in the square bracket on the right margin. **Maximum score is 20.**

===== **START OF QUESTIONS** =====

1. Consider the following data. [5]

Hours of air-condition usage per day	Monthly electricity bill in INR
2	4500
3	5000
5	7500
7	10000
9	13000

- (i) Find the equation of the line of best fit which minimises the least squares error.
 (ii) Based on your line of regression, estimate the monthly electricity bill with 8 hours of air-condition usage daily.

2. Consider a Markov chain $\{X_n\}_{n \geq 0}$ with states arranged on the vertices of a triangle. The transition probabilities between the states are as follows: $p_{12} = 1$, $p_{23} = \frac{1}{2}$, $p_{31}, p_{33} = \frac{1}{2}$, $p_{22} = \frac{1}{2}$. [5]

- (i) Construct the stochastic matrix \mathbb{P} .
 (ii) Find the stationary distribution of states $\vec{\pi}^{(\infty)}$.

3. Our friend, whose name is Daisy, hops around on a square pattern along its sides. At each step she moves to one of the neighbouring vertices at random (her decision is based on the flip of a fair two-sided coin). What is the expected number of steps taken by Daisy to get from vertex 1 to vertex 3?

Hint: assume that vertices 1 and 3 are diagonally apart. [5]

4. Safety features of a self-driving car are of paramount importance to manufacturers. This is the reason why modern autonomous cars have anywhere between 10 to 30 cameras on board. A certain self-driving car has three frontal cameras which are activated when the car gets close to a vehicle or object in front of the car. This is especially useful in a traffic jam and/or on the highway when a certain distance must be maintained between two successive cars. During foggy winter conditions, measurement of the exact distance of the car (d meters (m)) from the one in front, by a single input from a camera, may become less reliable. Therefore, the system must have built-in redundancies whereby multiple frames per second (fps) must be obtained successively by the three frontal cameras. Each of these measurements (all made every second) may be regarded to be an independent random variable with mean d m and standard deviation 2 m based on multiple statistical tests performed by the manufacturer. Then the average of all these measurements must be taken and processed per second by the electronic computer of the car to make the autonomous technology more accurate. What must be the fps rating of the frontal camera unit of the car so that the manufacturer is at least 95 % certain that the estimated information is accurate to within ± 0.5 m? [5]

===== **END OF QUESTIONS** =====