Instructions: Each group is assigned one problem. Within each group, you may work in teams but each of you must write and submit your own solutions. Violators may be penalized up to the maximum credit for the assignment. You must turn in your solutions as a scanned copy via email to `amriksen@thapar.edu` by 9 pm of May 27, 2020. Maximum credit for this assignment is five points.

1. **Group 1:**
   Show that all the roots of $P(z) = z^8 - 4z^3 + 10$ lie in the region $1 \leq |z| \leq 2$.

2. **Group 2:**
   Let D be a triangular region bounded by $x = 1, y = 1$ and $x + y = 1$. Find the image of D under the transformation $\omega = z^2$ and draw it. Is $\omega(z)$ a conformal transformation? Why?

3. **Group 3:**
   Use Rouche's theorem to show that $z + 3 + 2e^z$ has one root in the left half complex plane.

4. **Group 4:**
   State the argument principle. Use it to find the winding number of $f(z) := z^2 + z$ around $(0, 0)$ for each of the following curves:
   (i) C_1 is a circle of radius 2,
   (ii) C_2 is a circle of radius 1.

5. **Group 5:**
   Let $f(z)$ be a meromorphic function defined inside and on a Jordan curve C. Define the winding number in terms of $f(z)$ and justify why it is called the "winding" number?
   Hint: For the last part of the question you have to show that the appropriate integral evaluates to number of turns (or windings) of the contour in a suitable framework.

6. **Group 6:**
   State the Rouche's theorem. Use this theorem to prove the fundamental theorem of algebra.

$\square$