Linear algebra worksheet

December 13, 2021

1 RREF

1. If v_1,v_2,and v_3 are three linearly independent column vectors in R^4 then what will be the RREF of matrix A,where A is a matrix whose columns are above vectors .
2. Find the rank of the given matrix $\begin{bmatrix} 1 & 2 & 3 \\ 5 & 2 & 6 \\ 1 & 3 & 7 \end{bmatrix}$

2 Linearly dependent & independent vectors, basis of a vector space

3. Define the basis of a Vector Space and check whether the following column vectors forms a basis of R^4 or not.

   v_1=(1,1,0,0), v_2=(0,0,1,1),v_3=(1,0,0,4),v_4=(0,0,0,2)
4. Find a basis of R^{2*2}, the space of all 2×2 matrices, and thus determine the dimension of R^{2*2}
5. Find the basis of image space of matrix whose rows are given by (1,1,1),(1,2,5),(1,3,7). And also find the basis of kernel space.

3 Linear transformation

6. Let T from R^3 to R^3 be a function given by T(x,y,z)=(x,x+y,z) check whether T is linear transformation or not if yes then find its matrix representation.
7. Interpret the following linear transformation geometrically $T(X) = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} X$, where X is vector in R^2.
8. Find the basis of image space of matrix formed by column vectors in question (7).
9. Let T from R^3 to R^3 be linear transformation given by

   $T(e_1)$=(1,2,0), $T(e_2)$=(0,2,5),$T(e_3)$=(1,-1,-5) Then find its matrix representation.
10. For above given matrix representation find its corresponding linear transformation.
11. For matrix in question (6) verify rank-nullity theorem.
12. Let T from R^3 to R^3 be linear transformation given by T(1,0,0)=(1,2,3) ,T(0,2,0)=(1,0,0),T(3,1,1)=(0,1,0) find matrix representation of T

4 Gauss Jordan Elimination(Direct Method)

13. Consider the given system of linear equations

    x+y+z=4

    2x+2y+z=6

    x+y+2z=6 Is this system has a Unique solution or infinitely many solutions or no solution?

14. Solve the given system of linear equations

$$x_1 - x_2 + 3x_3 = 2$$
$$3x_1 - 3x_2 + x_3 = -1$$
$$x_1 + x_2 = 3$$

5 Iterative Methods

15. Find the first two iterations of Jacobi and Seidel method for the following system of linear equations using initial guess x=0,where $x = (x_1, x_2, x_3)$

    $3x_1 - x_2 + x_3 = 1$, $3x_1 + 6x_2 + 2x_3 = 0$, $3x_1 + 3x_2 + 7x_3 = 4$

16. Express the Jacobi iteration method for the linear system Ax = b given by

$$10x_1 - x_2 + 2x_3 = 6$$
$$-x_1 + 11x_2 - x_3 + 3x_4 = 25$$
$$2x_1 - x_2 + 10x_3 - x_4 = -11$$
$$3x_2 - x_3 + 8x_4 = 15$$

in the form $x^k = Tx^{k-1} + c$

6 LU Factorisation

17. Factor the following matrix into the LU decomposition using the LU Factorization Algorithm with $l_{ii} =$ 1 for all i. $\begin{bmatrix} 1 & -1 & 0 \\ 2 & 2 & 3 \\ -1 & 3 & 2 \end{bmatrix}$

18. Solve given system of linear equations using LU factorisation ,

$$x_1 - x_2 = 2$$
$$2x_1 + 2x_2 + 3x_3 = -1$$
$$-x_1 + 3x_2 + 2x_3 = 4$$

7 Gram Schmidt Orthogonalization

19. Find the QR factorization of the following matrix $\begin{bmatrix} 2 & 2 \\ 1 & 7 \\ -2 & -8 \end{bmatrix}$

20. Perform the Gram–Schmidt process on the following basis of R^2 and find an orthonormal basis, $\begin{bmatrix} -3 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ 7 \end{bmatrix}$