Mini-project: *Transactions on an accounting system*.

Shall we now bring some linear algebra into practice?

Objective: This simple mini-project will demonstrate an application from economics/accounting whereby we will be required to compute the basis of the null space of a certain matrix. This basis will represent the most fundamental unit of transaction in a closed accounting system.

Description: Consider a *closed* accounting system with n accounts, say $\alpha_1, \alpha_2, \alpha_3, \ldots \alpha_n$. At any instant, each account has a balance which can be a *credit* (positive), *debit* (negative), or *zero*. Since the accounting system must at all times be in balance, the sum of the balances of all the accounts will always be zero. Now suppose that a *transaction* is applied to the system. By this we mean that there is a flow of funds between accounts of this system. If as a result of the transaction the balance of account α_i changes by an amount t_i, then the transaction can be represented by an n-column vector with entries $t_1, t_2, t_3, \ldots, t_n$. Since the accounting system must still be in balance after the transaction has been applied, the sum of the t_is will be zero. The transactions correspond to column vectors of the form $\mathbf{t} = \begin{pmatrix} t_1 \\ t_2 \\ \cdot \\ \cdot \\ \cdot \\ t_n \end{pmatrix}$. Vectors of this form are easily seen to constitute a subspace $\mathbb{T}$ of the vector space $\mathbb{R}^n$; $\mathbb{T}$ is called the *transaction space.*

Questions:

Q1) Construct a matrix A such that $\mathbb{T}$ is just the null space of A.
Q2) Deduce the reduced row-echelon form, $\tilde{A}$ of A.
Q3) Consider the solution of the equation $\tilde{A}\mathbf{x} = 0$. Express $\mathbf{x}$ as a linear combination of the most canonical column vectors.
Q4) Deduce the basis of the *transaction space* $\mathbb{T}$.
Q5) What is the dimension of $\mathbb{T}$? Given that $\mathbb{T} \subset \mathbb{R}^n$, does the dimension of $\mathbb{T}$, you have just computed, make sense? Why?
Q6) Justify why your answer to Q4) above represents the most fundamental activity in this accounting system?