27/2/2019.

Lecture (10) Contd...

Th^m (Maximum principle)

(i) $f(z)$ analytic in $D \Rightarrow |f(z)|$ cannot have a max^m in D unless $f(z) = \text{Const}$.

(ii) $f(z)$ analytic in a bdd region D + $|f(z)|$ continuous in $\bar{D}$ $\Rightarrow |f(z)|$ attains its max^m on ∂D.

No proof req'd for this th^m. #

Consequences of above th^m:-

* Further if $f(z) \neq 0 \ \forall z$ w/in D

$\Rightarrow g(z) = \dfrac{1}{f(z)}$ is s.t. $|g(z)|$ attains its maximum on ∂D

i.e. $f(z)$ attains its minima on ∂D.

This is a very important result in the theory of PDEs & in applied math in general

* The max^m principle implies that the harmonic f^n_s achieve their maxima (& minima) on the boundary of the regions

How/Why?

$f(z) = u(x,y) + iv(x,y)$ analytic (satisfy CR eqns & are infinitely differentiable in the complex plane b/c of the derivative version of Cauchy integral formula)

$\Rightarrow g(z) = e^{f(z)}$ is analytic

$\therefore |g(z)| = e^{u(x,y)}$ attains its max^m on ∂D (b/c of Max^m principle (ii))

further $h(z) = e^{-if(z)}$ is analytic $\Rightarrow |h(z)| = e^{v(x,y)}$ attains its max^m on ∂D (b/c of same reason)

$\Rightarrow u(x,y), v(x,y) \in \mathbb{R}$ attain their max^m on ∂D. #.

Likewise for minima...