HW(1) 9/8/18 pg①

Q.1. (Law of total probability, Conditional probability).

The game of roulette has many versions:-

Version (1): (Las Vegas Roulette) has 38 numbers, 0 and 00 are painted green & 1 to 36, half of them being red & rest painted black. If you bet on red; the probability to lose is 20/38.

Version (2): (Monte Carlo Roulette I) has 37 numbers, 0 and 1 to 36 half of them being red & remaining half being black. If you roll a 0 then you are sent to prison (P1). At the next spin if you get a red you get your bet back (and nothing more); if you get black or 0, you loose.

(i) What is the probability to win?
(ii) What is the probability to loose?

Version (3): (Monte Carlo Roulette II) is played as in version (2) but with a second prison (P2). If you are in the first prison P1, you loose if the next spin is black & if the next spin is a 0, you are sent to prison P2. In P2 you loose if you get a black

or 0 and are sent back to P1 if you get a red.
Compute your loss probability ?

Q2) Bayes thm application. (Witness reliability)

After a robbery the thief jumped into a taxi & disappeared. An eyewitness on the crime scene is telling the police that the cab is yellow. In order to make sure that this testimony is worth something the assistant police officer makes a Bayesian analysis of the situation. He comes up with the following information:

→ In that particular city, 80% of taxi's are black & 20% of taxis are yellow.

→ Eyewitness are not always reliable & from past experience it is expected that an eyewitness is 80% accurate; i.e. he will identify the color of a taxi accurately (yellow or black) 8 out of 10 times.

Based on this above information, predict if the information provided to the assistant police officer by the witness of the robbery is of any use.

{Hint :- find Prob(Color of taxi = yellow | reported color is yellow)}

Q.3) The Monty's Hall problem. (Tol Mol ke Bol ??)

At a game show the host hides a prize (say ₹ 1 lakh) behind one of 3 doors and nothing of much value behind the remaining two doors (in the usual story 2 goats !!). The contestant picks one of 3 doors, let us say door 1 & the host opens one of the remaining doors (say door 3) which reveals a goat. The contestant is then given the choice to either switch to door 2 or keep door 1. What should he do?

Q.4) The geometric distribution of type 1 for X,

$X \sim \text{geom}_1(p)$:= no. of attempts until first success.

Refer to your class notes to compute $E(X)$. Now, using similar ideas, find $Var(X)$.

Q.5) geometric D^n

I am new to basketball and so I am practicing to shoot the ball through the basket. If the probability of success at each throw is 0.2; how many times would I fail on avg. before I taste success.

Q.6) Using the def^n. of conditional probability, show that the geometric distribution exhibits memoryless property i.e.

$$P(X \geq i+j \mid X \geq i) = P(X \geq j)$$

Q.7. Show that the Poisson distribution is a limiting case of the Binomial distribution when $n \to \infty$ but rate $\lambda = np$ is fixed.

Q.8. (Mean time to failure) It is a critical parameter in the design of parts in a system. Let us suppose that the ball-bearing mechanism in a rotating machine fails at the end of each hour of use with probability 0.4, if it has not failed already. What is the life expectancy of the ball-bearing mechanism?

You may solve this problem in one of 2 ways:-

(i) Using $E(C) = \sum_{i=1}^{\infty} i\,Prob(C=i) = \sum_{i=0}^{\infty} Prob(C>i)$ — (8.1)

where C is the no. of hours to failure. First prove/show why (8.1) is true.

(ii) Using the law of total expectation (similar to our problem of squirrel in a random walk).

Recall from lecture that

$$E[I_A] = Pr(A) \quad ; \text{ where } I_A \text{ is an indicator R.V.}$$

Q.9.) (Application of Indicator R.V. & computing expected values).

Suppose there is a dinner party, where 100 men check their hat's (i.e. turn in their hats at the counter before dinner). During dinner, the hats are mixed up. Clearly, each man gets his own hat with probability 1/100 after dinner. What is the expected no. of men who get their own hat?

Q.10 (The coupon collector problem).

There are 10 different types of coupons (each of them labelled with a different color). The type of coupon awarded to us each day by the kind woman at the "Easy Day grocery" register is selected uniformly & independently at random. What is the expected no. of visits to the grocery store in order to acquire atleast one of ~~exactly~~ each type of coupon?