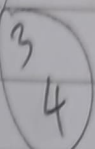
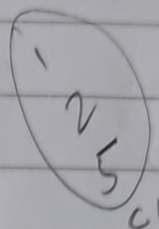
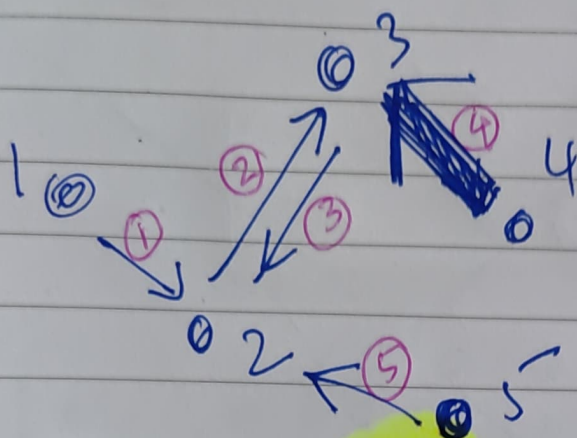


Prog. Assessment. (LI)



c2

→ Arrows.

Citizens ↓

$$M = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 2 & 1 & -1 & 1 & 0 \\ 3 & 0 & 1 & -1 & 1 \\ 4 & 0 & 0 & 0 & -1 \\ 5 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Remove

$$\text{rref}(M) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Pivots are identified

$$M_R = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 4 & 5 \\ -1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

→ reduced the original matrix M

$$M_{RT} = (M_R)^T = \begin{pmatrix} -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & 0 & -1 \end{pmatrix} \rightarrow \text{citizens}$$

Communities: Null sp. (M_{RT})

(S) Why should you have guessed this?

Ans) We recall the discussion on tandem disjoint accounts!

$$\begin{pmatrix} -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & 0 & -1 \end{pmatrix}_{3 \times 5} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix}_{5 \times 1} = \vec{0}$$

$$\Rightarrow -x_1 + x_2 = 0 \Rightarrow x_2 = x_1$$

$$x_3 - x_4 = 0 \Rightarrow x_3 = x_4$$

$$x_2 - x_5 = 0 \Rightarrow x_5 = x_2$$

$$\left. \begin{array}{l} \text{i.e. } x_1 = x_2 = x_5 \\ \text{and } x_3 = x_4 \end{array} \right\} \text{i.e. } \vec{x} \in \text{null}(M_R)$$

$$\vec{x} = \begin{pmatrix} \alpha \\ \alpha \\ \beta \\ \beta \\ \alpha \end{pmatrix}$$

$$\bar{X} = \alpha \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} + \beta \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{pmatrix}$$

Basis of Null (MRT)
Prescribe the communities

1
2
5

C¹

3
4

C²

#