Continuous Probability Distributions

1) Normal (or Gaussian) D^n.

$$X \sim N(\mu, \sigma^2)$$

$$f_X(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad ; \sigma > 0$$

$$E(X) = \mu$$

$$Var(X) = \sigma^2$$

2) Uniform D^n

$$X \sim \text{uniform}(a, b)$$

$$f_X(x) = \frac{1}{b-a} \quad ; \quad a \le x \le b$$

$$E(X) = \frac{b+a}{2} \quad ; \quad Var(X) = \frac{(b-a)^2}{12}$$

3) <u>Pareto D^n</u> ($\mathcal{D}^n$ of wealth in a society - fitting the trend that a large portion of wealth is held by a small fraction of the population).

$X \sim \text{Pareto}(\alpha, \beta)$

α: scale, β: shape

$$f_X(x) = \frac{\beta \alpha^\beta}{x^{\beta+1}} \quad ; \; x > \alpha, \quad \alpha, \beta > 0$$

$$E(X) = \frac{\beta \alpha}{\beta - 1} \quad ; \; \beta > 1 \quad (\infty \text{ otherwise})$$

$$Var(X) = \frac{\beta \alpha^2}{(\beta-1)^2(\beta-2)} \quad ; \; \beta > 2$$

4) <u>Exponential D^n.</u> (waiting times)

$X \sim \exp(\theta)$; $\theta \sim \frac{1}{\lambda}$; where λ is rate (think Poisson D^n).

$$f_X(x) = \frac{1}{\theta} e^{-x/\theta} \quad ; \; x \geq 0 ; \; \theta > 0 \quad ; \quad E(X) = \theta, \; Var(X) = \theta^2$$

5) Gamma D^n (Waiting time D^n).

* generalization of exponential & χ^2 D^n

$$X \sim \Gamma(\alpha, \beta)$$

shape (α), scale (β)

$$f_X(x) = \frac{1}{\Gamma(\alpha)\beta^\alpha} x^{\alpha-1} e^{-x/\beta}$$

$$E(X) = \alpha\beta$$

$$Var(X) = \alpha\beta^2$$

6) Other examples of continuous D^n. :- χ^2, t_ν, F_{ν_1,ν_2}

these are sampling D^n.

Joint D^n and Marginal D^n. (for multivariate D^n).

$$P(X=x, Y=y) = P(Y=y \mid X=x)\underbrace{P(X=x)}_{\text{Marginal } D^n}$$

⟷ this is analogous to cond'al prob. $P(A|B) = \frac{P(A \cap B)}{P(B)}$

Where $\int_y \int_x P(X=x, Y=y) = 1$ (continuous case) → will discuss this below

OR $\sum_i \sum_j P(X=x_i, Y=y_j) = 1$ (Discrete case).

Continuous case:

$$f_{XY}(x,y) = f_{Y|X}(y|x)\underbrace{f_X(x)}_{\text{marginal pdf}} = f_{X|Y}(x|y)\underbrace{f_Y(y)}_{\text{marginal pdf}}$$

Where $\int_x \int_y f_{XY}(x,y) = 1$

So far we have discussed examples of simple RVs.

Now! Some examples of D^n of composite RVs.

Q) Let X & Y be independent geom. (p)

(a) Find the D^n $\min(X, Y)$

(b) $P(Y \geq X)$

(c) Find D^n of $X + Y$

(d) $P(Y = y \mid X + Y = z)$ for $z \geq 2$; $y = 1, \ldots, z-1$

Soln:- (b) $P(Y \geq X) = \sum_{x=1}^{\infty} P(X = x, Y \geq X)$ $\overset{\text{think}}{\longleftrightarrow}$ $P(X=1, Y \geq X) + P(X=2, Y \geq X) + \cdots P(X=\infty, Y \geq X)$

$= \sum_{x=1}^{\infty} P(X = x, Y \geq x)$

$P(X=x, Y=y) = P(X=x) \cdot P(Y=y)$ when X & Y are independ.

$\overset{\text{indep}}{=} \sum_{x \geq 1} P(X = x) P(Y \geq x) = \sum_{x=1}^{\infty} p(1-p)^{x-1} (1-p)^{x-1}$

$= p \sum_{x=1}^{\infty} (1-p)^{2(x-1)} = \frac{p}{2p(1-p)}$