School of Mathematics, TIET, Patiala

Name: —————————————————————————————————————-

Roll Number: ———————————————————————————————

Instructor's name of your section: ————————————————————

Total Time: 2 hours
Maximum Score: 25

Instructions: Attempt all questions. Credit will be awarded in half-integer multiples only. Appropriate steps must be shown in order to receive any credit for your solutions. Points allotted for each question is shown in square parenthesis. This question paper has two pages. Please attach and return the question paper along with your solution booklet.

1. **Basics of Probability and Distributions**

    (a) Define Bernoulli trials.   [1]

    (b) Each sample of water has a 10% chance of containing a particular pollutant. Assume that the samples are independent with regard to the presence of the pollutant.
    i) Find the probability, that in 18 samples of water, exactly 2 contain the pollutant.   [1]
    ii) Find the probability that at least 4 samples contain the pollutant.   [1]

    (c) In a certain assembly plant, three machines, M1, M2, and M3, make 30%, 45%, and 25% of the products respectively. Past experience shows that 2%, 3%, and 2% of the products made by the respective machines are defective. Suppose that a final product is randomly chosen what is the probability that it is defective? If the product was chosen randomly and found to be defective, what is the probability that it was made by machine M3?   [3]

    (d) Consider the following table which provides the joint probability distribution of two discrete random variables viz., gender and smoking habit.

		smoking habit	
		smoker	non-smoker
gender	male	0.65	0.1
	female	0.1	0.15

    i) Find the probability that a person be female given that she is a smoker.   [1]
    ii) Find the probability that the person is non-smoker given that the person is female.   [1]

    (e) A smart phone shop owner observed that the number of smart phones sold over the past 100 days is given in the following table.

number of phones sold	days
0	8
1	12
2	30
3	40
4	7
5	3

    i) Find the probability of selling more than 3 smartphones in a day.   [0.5]
    ii) Compute the expected number of phones sold in a day.   [0.5]
    iii) Compute the variance for the same.   [2]

(f) (*Gambler's ruin*) A gambler starts with Rs x. She tosses a *fair* coin repeatedly. Outcomes of each toss are independent of all previous outcomes (tosses). If she gets a <u>Head</u>, she wins Re 1. If she gets a <u>Tail</u>, she loses Re 1. She continues to gamble until she has either Rs N or no money left to gamble. What is the probability of the gambler's ruin, i.e. she ends up losing all her fortune?   [5]

<u>Hint</u>: Say, R := the event that the gambler is ruined. $p_x := Prob(R|$ starting with Rs $x)$, $\forall x = 0, 1, 2, ..., N$. The boundary conditions are the values of p_0 and p_N. For properly identifying the values of the boundary conditions, you will be credited with two points. For appropriately partitioning the sample space and conditioning on the win and loss events for every toss, you will be awarded one point. For finding the correct recurrence relation, you will receive one point. For solving the recurrence relation, you will receive one point. The solution of the recurrence relation may be obtained in multiple ways, eg., you may use the theory of solving difference equations. However, you may resort to a simpler repeated substitution method to obtain the solution. If you chose to use the latter method, begin by writing an expression for p_2 in terms of p_1 by substituting the boundary condition for p_0 in the recurrence relation. Then find p_3 in terms of p_1 by substituting p_2 from the previous step. Likewise, obtain p_x in terms of p_1, which itself can be found by using the boundary condition for p_N in the expression for p_N.

2. **Discrete time Markov chains**

   (a) State the Chapman-Kolmogorov equation.   [1]

   (b) Assume there are three telecom companies operating in Punjab viz., Jio, Airtel, and Vodafone. Further, assume that 10,000 people use these companies in Thapar University Campus during any month of which 3,000 people use Jio, 4,000 use Vodafone and 3,000 use Airtel. A survey conducted by a consultancy firm indicates that 75% of the people who currently use Jio will prefer to use Jio in the next month. However, 15% of them will switch to Vodafone while 10% of them will prefer to use Airtel in the next month. Among subscribers who currently use Vodafone, 70% will prefer to use Vodafone in the next month while 15% will switch to Jio in the next month. Among current Airtel subscribers, 65% will prefer to use Airtel in the next month while 15% will switch to Jio in the next month.

   (i) Construct the initial distribution and probability transition matrix.   [2]
   (ii) Determine the market shares of these telecom companies after 3 months.   [2]

   (c) Our bunny, whose name is Honey, hops around on a triangle. At each step he moves to one of the other two vertices at random (his decision is based on the flip of a fair two-sided coin). What is the expected time taken by Honey to get from vertex 1 to vertex 2?   [4]

   <u>Hint</u>: Identification of the state space will fetch you one point. Construction of the appropriate probability transition matrix will get you one point. Computing the expected hitting time will fetch you two points, one point for writing the correct set of equations for the mean hitting time and one point for the correct numerical value.

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