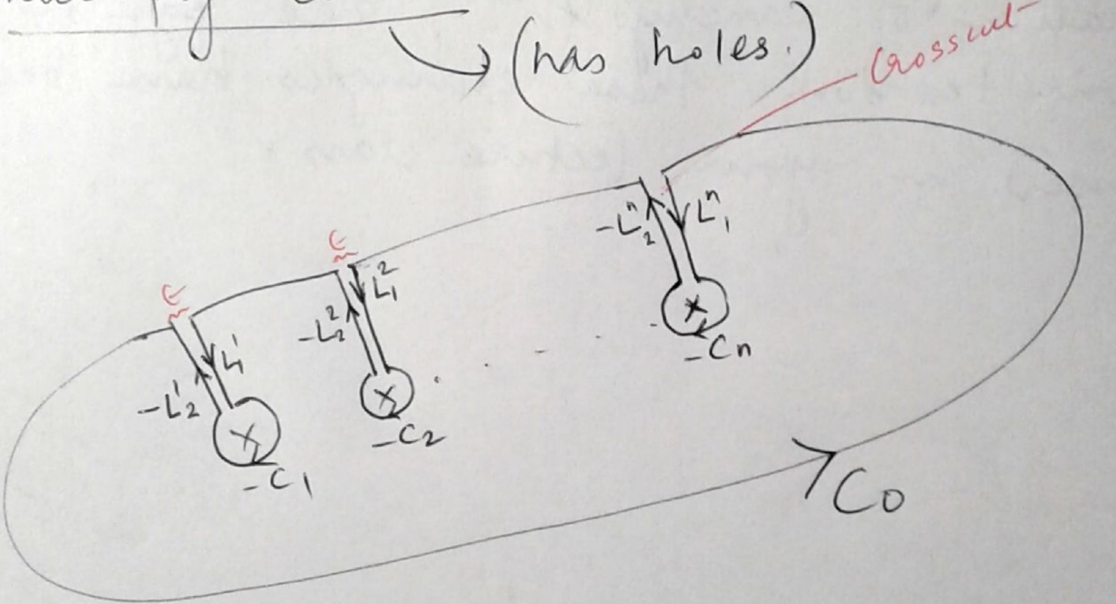
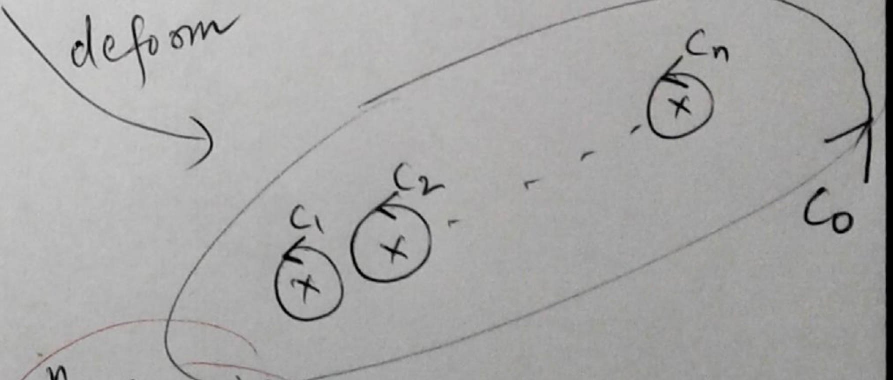
Lecture (10): Deformation of contour & application of Cauchy - Goursat Th^m.

A simply connected domain is one where one can continuously shrink any simple closed curve into a pt. while remaining in the domain.

In $\mathbb{C}$ or in 2D, it is a domain w/o holes.

A domain that is not simply connected is multiply connected. → (has holes.)





$$\tilde{C} = C_0 - \sum_{j=1}^{n} C_j + \sum_{j=1}^{n} (L_1^j - L_2^j)$$

→ Contribution of this to $\int f(z)\,dz = 0$ in the limit $\epsilon \to 0$

So in the limit $\oint_{\tilde{C}} = \oint_{C_0} + \sum_{j=1}^{n} \oint_{-C_j}$ & $\oint_C f(z)\,dz = 0$ if $f(z)$ is analytic.

$$\Rightarrow \oint_{C_0} f(z)\,dz = \sum_{j=1}^{n} \oint_{C_j} f(z)\,dz$$

w/ all contours C_0 and C_j taken in counterclockwise sense.

We say C_0 has been deformed into the contours C_j; $j = 1, 2, \dots, n$.

* for examples on contour deformation & Application of Cauchy's thm ; see pgs. 87-89 of your textbook. These examples have been discussed in your lecture class.