(1)

More thm's & proofs on fixed pt. iteration.

Thm (2.3) Let $g \in C[a,b]$ be s.t. $g(x) \in (a,b) \ \forall x \in [a,b]$

Further, g' exists on (a,b) & a constant $0 < K < 1$ exists w/ $|g'(x)| \le K \ \forall x \in (a,b)$

then, for any p_0 in $[a,b]$;

the seq. $p_n = g(p_{n-1})$; $n \ge 1$

converges to the unique f.p. p in $[a,b]$

Proof :- Do it yourself (ref. pg 59 of text book by Burden & Faires, 9th edn.)

very similar to nxt. proof!

Also, Very similar to proof of thm (2.2)! (You just have to apply the MVT ineq. mul. times)

Corollary (2.4) :- If $g(x)$ satisfies the hypotheses of th^m (2.3); then bounds for the error involved in using p_n to approximate p are given by

(i) $|p_n - p| \le K^n \max\{p_0 - a, b - p_0\}$;

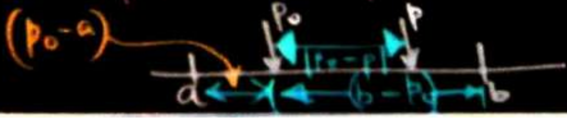
and (ii) $|p_n - p| \le \frac{K^n}{1-K} |p_1 - p_0|$; $\forall n \ge 1$.

Proof :- $\because p \in [a, b]$;

$$|p_n - p| = |g(p_{n-1}) - g(p)| \overset{MVT}{=} g'(\xi_n)|p_{n-1} - p| \le K|p_{n-1} - p|$$

where $\xi_n \in (a, b)$

induction $\Rightarrow$ (i) $|p_n - p| \le K|p_{n-1} - p| \le K^2|p_{n-2} - p| \le \cdots \le K^n|p_0 - p|$
$\le K^n \max\{p_0 - a, b - p_0\}$



Also, $|p_{n+1} - p_n| = |g(p_n) - g(p_{n-1})| \le K|p_n - p_{n-1}| \le \cdots \le K^n|p_1 - p_0|$ $\boxed{(3)}$

(ii) thus, for $m > n \ge 1$

$$|p_m - p_n| = |p_m - p_{m-1} + p_{m-1} - \cdots + p_{n+1} - p_n|$$

$$\overset{\Delta\text{-ineq}}{\le} |p_m - p_{m-1}| + |p_{m-1} - p_{m-2}| + \cdots + |p_{n+1} - p_n|$$

$$\le K^{m-1}|p_1 - p_0| + K^{m-2}|p_1 - p_0| + \cdots + K^n|p_1 - p_0|$$

$$= K^n|p_1 - p_0|(1 + K + K^2 + \cdots + K^{m-n-1})$$

$$|p - p_n| = \lim_{m\to\infty}|p_m - p_n| \le \lim_{m\to\infty} K^n|p_1 - p_0|\sum_{i=0}^{m-n-1} K^i$$

$$\le K^n|p_1 - p_0|\sum_{i=0}^{\infty} K^i$$

$$= \frac{K^n}{1-K}|p_1 - p_0|$$

#