Concepts: Compare the calculation of case (I) & case (II) below!

(I) Recall the strategy for discrete RV (i.e. X, Y discrete RV).

$$P(X+Y \le z) = P(Y \le -X + z)$$

partition X over all possible disjoint outcomes of X i.e. all possible values x can take.

$$= \sum_{x=1}^{\infty} P(X=x, Y \le -X+z)$$

$$= \sum_{x=1}^{\infty} P(X=x, Y \le -x+z)$$

$$\overset{\text{indep.}}{=} \sum_{x=1}^{\infty} P(X=x)\, P(Y \le \underbrace{-x+z})$$

think of this as y

(II) $$P(Y \le y) = \int_{-\infty}^{y} f_Y(y)\,dy = \int_{-\infty}^{y} \left(\int_{-\infty}^{\infty} f_{XY}(x,y)\,dx \right) dy$$

$$= \int_{-\infty}^{y} \left(\int_{-\infty}^{\infty} f_X(x)\,dx \right) f_Y(y)\,dy$$

$$= \int_{-\infty}^{\infty} \left(\int_{-\infty}^{y} f_Y(y)\,dy \right) f_X(x)\,dx$$

b/c $f_{XY}(x,y) = f_X(x)\, f_Y(y)$

Compare!

$y = -x + z$

Discrete Case:- $P(X+Y \le z) = P(Y \le -X + z)$

$$= \sum_{x=1}^{\infty} P(X = x) P(Y \le y)$$

Continuous case:-

$$P(Y \le -X + z) = \int_{-\infty}^{\infty} \left(\int_{-\infty}^{y} f_Y(y)\,dy \right) f_X(x)\,dx.$$

Here also $y = -x + z$