UMA 011
Numerical Analysis.

→ What is this course about ?

Textbook: Numerical Analysis by Burden & Faires

→ Theory + Lab.

→ Evaluations → 70 % (MST + EST)

→ 30 % (Lab + sessional)

Generally

Online Mode.

Awaiting DOAA's instructions

Lab → Continuous Eval^{n} (5%) + Viva (5%)

Matlab based numerical experiments

→ All notes + Zoom recording of Lectures will be available on my webpage :- www.amriksen.com → then go to teaching tab

→ All general instructions relating to Course (exams, quiz, Lab experiments, etc) will be available through LMS.

from §(1.2) Chopping & rounding

→ Computer is a finite state m/c w/ finite memory

→ Electronic hardware – on & off states – 0, 1 (bin representⁿ) of (digital) no.s.

→ Decimal floating pt. representation of any +ve real no. can be represented as

$$y = 0.\underbrace{d_1 d_2 \cdots d_k d_{k+1} d_{k+2} \cdots}_{\text{Mantissa / fractional part}} \times 10^n \quad ; \quad 1 \le d_1 \le 9, \; 0 \le d_i \le 9 \;\forall\; i = 3, \cdots, k$$

→ Computer has finite memory → Chop($0.d_1 d_2 \cdots d_k \times 10^n$) (kth decimal place)

→ round($0.\delta_1 \delta_2 \cdots \delta_k \times 10^n$)

eg. Express 0.7153987×10^6 upto 5 decimal places.

(i) Chopping :– 0.71539×10^6

(ii) rounding :– 0.71540×10^6

§(1.2) Error (in representation) & Significant digits.

p is the actual true value; p^* is approximate num. rep^n.

Absolute error :- $|p - p^*|$

Relative error :- $\frac{|p - p^*|}{|p|}$; $p \neq 0$

Significant digits :-

eg $\underline{7}3\underline{0}0$

$0.00\underline{65}$

$\underline{3020}.\underline{40}$

$\underline{8}\underline{0}\underline{0}\underline{2}$

} Underlined digits are significant.

So what are the rules to identify significant digits?

Rules for finding Significant digits (s.d.)

1) All non-zero no.s ARE significant. eg. 33.2 has 3 s.d.

2) Zeros between two non-zero digits ARE significant-
eg. 2051 has 4 s.d.

3) Leading zeros are NOT significant.
eg. 0.0032 has 2 s.d.

4) Trailing zeros to the right of the decimal pt. ARE significant eg. 92.00 has 4 s.d.

Why? B/c an experimenter who estimates (calculates) 92.00 kg/m³ to be the density of a substance X knows the estimate is accurate up to 2 decimal pts. whereas another experimentalist who estimates the same value to 92 kg/m³ has done so less precisely i.e. does not know the accuracy after decimal pt.

(5) Trailing zeros in a whole no. with decimal shown ARE significant eg. 540. has 3 s.d.

(6) Trailing zeros in a whole no. with no decimal shown are NOT significant. eg. 540 has 2 s.d.

(7) Exact no.s have an INFINITE no. of s.d.
eg. 1 meter = 1.00 meters = 1.0000 meters = 1.000000000 meters

(8) For a no. in scientific notation: $N \times 10^x$, all digits comprising N ARE significant by the first 6 rules; "10" and "x" are NOT significant

NOTE:- rule (8) provides the opportunity to change the no. of s.d. in a value by manipulating its form. eg. 1100 has 2 s.d.
1100. has 4 s.d.
1.10×10^3 has 3 s.d.

Question:- Express 1000.3 in a form w/ 4 s.d.

Ans:- 1000.3 has 5 s.d.

Rounding gives 1000 has 1 s.d.

Now express it as 1000. w/ 4 s.d.

Question Let $p = 0.54617$, $q = 0.54601$. Assume you have a computer which has a precision s.t. it can accurately handle 4-digit arithmetic. Which among rounding & chopping will give you better precision to calculate $r = p - q$?

Ans:-

Rounding

$p^* = 0.5462$, $q^* = 0.5460$

$r^* = p^* - q^* = 0.0002$ has 1 s.d.

$$\text{rel. error} = \frac{|r - r^*|}{|r|} = \frac{|0.000016 - 0.0002|}{|0.00016|} = 0.25$$:)

chopping

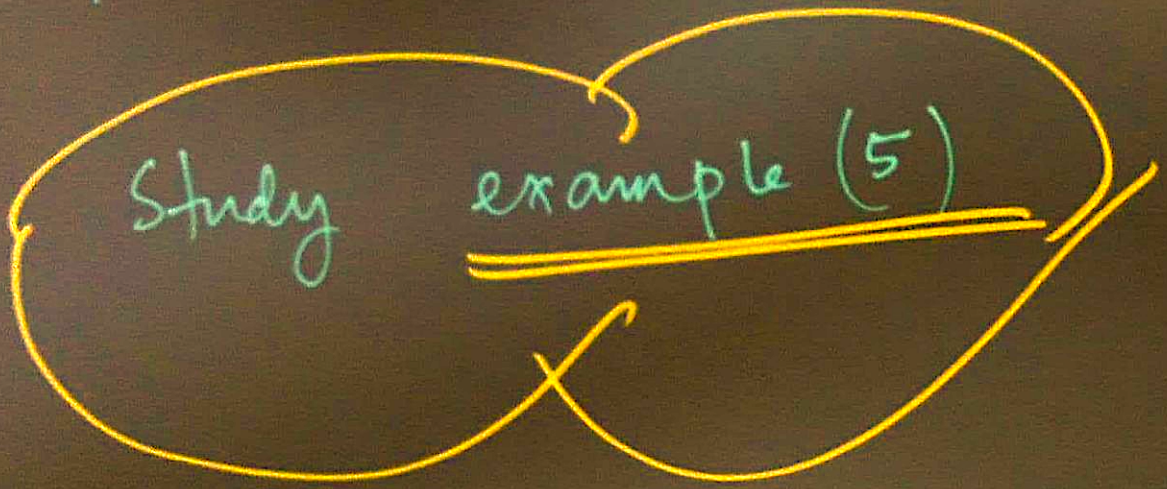
$p^* = 0.5461$, $q^* = 0.5460$

$r^* = p^* - q^* = 0.0001$ has 1 s.d.

$$\text{rel. err} = \frac{|0.00016 - 0.0001|}{|0.00016|} = 0.375$$:(

Reading assignment!

from text book – "Numerical Analysis" by Burden & Faires §(1.2) on pg. 23-24



this example shows you how to use a certain "rationalization" technique in a formula to increase the accuracy of numerical computation in a finite state m/c (computer)!