PLAKSHA UNIVERSITY, SPRING SEMESTER AY 2024-25

Course Code: FM126

Course Title: Computational Linear Algebra

Course Credits: 03 **L / T / P: 2/0/1**

Course intended for: 2nd semester undergraduate students.

Prerequisites: College algebra, elementary familiarity with vectors and matrices.

Class Schedule: L1: T, Th 12:00 PM – 12:50 PM (2201),

L2: T, Th 10:30 AM – 11:20 AM (2202).

Classroom No.: 2201, 2202.

Lab Schedule: L1P1: T 02:00 PM – 03:50 PM (1101),

L1P2: T 02:00 PM – 03:50 PM (1102),

L2P1: Th 02:00 PM – 03:50 PM (1101),

L2P2: Th 02:00 PM – 03:50 PM (1102).

Lab room number: L1P1: 1101, L1P2: 1102, L2P1: 1101, L2P2: 1102.

Lead Instructor: Amrik Sen

Office hours: Room no. A2-103

Email ID: amrik.sen@plaksha.edu.in

Teaching Fellows: Mr. Rajat Singla, Mr. Vijay Sahani, Ms. Sakshi Jaiswal

Course Description

This course will cover fundamental aspects of linear algebra from the stand point of basic theoretical knowledge and practical applications. Students will learn how to use a computer and a programming framework to solve mathematical problems relevant to a broad engineering curriculum. Several practical examples will be discussed that will enable students to anchor abstract concepts to relatable applications.

Course Overview

This course provides a comprehensive exploration of linear algebra, focusing on its applications in engineering systems and computational techniques. It begins with foundational concepts such as matrix operations, vector spaces, and systems of linear equations and covers some advanced topics on computational methods like matrix decompositions (LU, QR, Cholesky) and eigenvalue problems. Real-world applications like the Google Page Rank algorithm, large language models, and financial accounting systems will be discussed.

Learning Objectives

1. The course will familiarize students with the fundamentals of linear algebra such as matrix operations, vector spaces, and solving systems of linear equations.
2. The course will train students to apply matrix decomposition methods (LU, QR, Cholesky) and eigenvalue techniques to solve engineering problems.
3. The course will equip students with the skills to implement solutions for linear systems using Python.
4. The course will enable students to solve real-world problems such as banking systems and the Google Page Rank algorithm using computational techniques.
5. The course will train students to model dynamical systems as eigenvalue problems and obtain their solutions.

Learning Outcomes:

By the end of the course, students will be able to:

1. Construct matrix representations of systems of linear equations and find their numerical solutions.
2. Compute orthonormal bases of vector spaces both analytically and numerically using a computer algorithm.

3. Compute eigenvalues and eigenvectors of matrices analytically and numerically.
4. Perform linear transformations (change of basis) of simple models.
5. Use concepts in matrix algebra to frame engineering and scientific problems in mathematical language and obtain their solutions.

Recommended Textbook

I. **Linear Algebra with Applications** by *Otto Bretscher*, fifth edition, Pearson, (2010).

Additional Reference Textbooks

I. **A Course in Linear Algebra with Applications** by *Derek J. S. Robinson*, second edition, World Scientific, (2006).
II. **Differential Equations and Linear Algebra** by *Jerry Farlow et. al.*, second edition, Pearson, (2007).

Assessment Components:

- **Term Exams: 50%**
  - Mid-semester exam (20%) Week-9 (Mar 08 - 13, 2025)
  - Final comprehensive exam (30%) Week-18 (May 09 - 15, 2025)
- **Assessment for coding and computation: 10%**
  - All groups Week–8 (L1P1, L1P2: Mar 04, 2025, L2P1, L2P2: Mar 06, 2025)
- **Interviews: 10%**
  - All groups Dates as assigned below in the weekly class plan
- **MCQ quizzes: 20%**
  - Quiz 1 – 10% Week–5 (L1P1, L1P2: Feb 11, 2025, L2P1, L2P2: Feb 13, 2025)
  - Quiz 2 – 10% Week–16 (L1P1, L1P2: Apr 29, 2025, L2P1, L2P2: May 01, 2025)
- **Attendance: 10%** (mandatory for all UG courses as per university policy)

  >80% = **10 points**
  70 to 80 % = **8 points**
  60 to 70 % = **6 points**
  < 60 % = **0 points**

Weekly Class Plan

Date	Lecture Topics	Laboratory
Week 1 (Jan 13-17, 2025)	Introduction 1 (syllabus)	Review of Python library NumPy
Week 2 (Jan 20-24, 2025)	Introduction 2 – Application problem 1: Motivation to the course: systems of linear equations, examples from mechanical and electrical engineering systems (oscillators: spring-mass, RLC circuits) Writing systems in matrix vector forms, types of matrices depending on system specifications	Workshop on visualization of mathematical functions and models using Python
Week 3 (Jan 27-31, 2025)	Introduction to vector spaces Matrix forms, basic matrix operations	Workshop on matrix operations using Python
Week 4 (Feb 03-07, 2025)	Linear independence, spanning vectors, basis of a vector space; elementary idea of a subspace Geometric insights on vector spaces	➢ Vectors and their geometric properties using Python ➢ Practice worksheet 1
Week 5 (Feb 10-14, 2025)	Row space, column space, null space (kernel) Echelon form, pivots, rank of a matrix	➢ MCQ Quiz-1 (to be held in lab class) ➢ Practice worksheet 2
Week 6 (Feb 17-21, 2025)	Rank-nullity theorem, four fundamental subspaces of a matrix Application problem 2: Banking and accounting system	➢ Vector spaces using Python ➢ Practice worksheet 3

Week 7 (Feb 24-28, 2025)	Linear transformations (change of basis) Matrix norms, convergence (basic ideas)	➢ Interview – Group 1 ➢ Practice worksheet 4
Week 8 (Mar 03-07, 2025)	First pass on solving systems of linear equations - consistent and inconsistent systems Augmented matrix, solving systems - Gauss elimination (by hand)	Assessment for coding and computation using Python
Week 9 (Mar 10-14, 2025)	Mid-semester exam	Mid-semester exam
Week 10 (Mar 17-21, 2025)	Gauss elimination (programming method, issues with pivots, etc) Elementary ideas on iterative methods, comparison with direct methods	➢ Solving systems of equations using Python ➢ Practice worksheet 5
Week 11 (Mar 24-28, 2025)	Gram-Schmidt orthonormalization, constructing subspaces Matrix decompositions - LU, QR	➢ Interview – Group 2 ➢ Practice worksheet 6
Week 12 (Mar 31 – Apr 04, 2025)	Matrix decompositions - L+D+U, Cholesky Introduction to eigenvalues and eigenvectors, examples, physical significance and geometric meaning	➢ Interview – Group 3 ➢ Practice worksheet 7
Week 13 (Apr 07-11, 2025)	Characteristic equations, Caley-Hamilton result, problems on ev and EV	➢ Construction of vector subspaces and matrix decompositions using Python ➢ Practice worksheet 8

Week 14 (Apr 14-18, 2025)	Numerical technique to solve ev_max (Power method) ev_min and other eigenvalues (inverse, shifted power method)	➢ Interview – Group 4 ➢ Practice worksheet 9
Week 15 (Apr 21-25, 2025)	Application problem 3: Google page rank algorithm (application of the power method and solving systems of linear equations) — connection with Math of Uncertainty (Stochastic matrix) Application of evs, EVs, subspaces to solve a system of ODE (only paper pencil method) — superposition principle	➢ Interview – Group 5 ➢ Practice worksheet 10
Week 16 (Apr 28-May 02, 2025)	Similarity transformation (diagonalisable matrices) and examples Spectral decomposition and examples	➢ MCQ Quiz-2 (to be held in lab class) ➢ Practice worksheet 11
Week 17 (May 05-09, 2025)	Application problem 4: Large language models with matrices and vector spaces Normal matrices, singular values, SVD *(if a lecture slot is available)*	Residual assessment (students may replace any one of their class assessments, except the mid-term and end-term exams, with this optional test; any missed assessment may also be considered)
Week 18 (May12-16, 2025)	End-semester exam	End-semester exam

PLAKSHA ACADEMIC POLICY (WILL BE PROVIDED BY OAA)

Makeup Policy

Attendance Policy

Policy on Incompletes

Scholastic Misconduct

University Policy (will be provided by academic office)

Graphical Abstract

Did you know that your money lives in a matrix?