Tutorial Worksheet-4 (WL5.1, WL5.2)

Orthogonal basis, properties of Orthonormal vectors, orthogonal projection and orthogonal complement, properties of orthogonal complement, advantage of orthogonal transformations, Gram-Schmidt process

Name and section: ______________________________

Instructor's name: ______________________________

1. Find the orthogonal projection $\vec{x}^{\parallel} = proj_v(\vec{x})$ of the vector $\vec{x} = (1, 2, 3)^T$, onto vector $\vec{v} = (-1, 0, 1)^T$.

2. Find the orthogonal projection of $\begin{bmatrix} 9 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ onto the subspace of $\mathbb{R}^4$ spanned by $\left\{ \begin{bmatrix} 2 \\ 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 2 \\ 0 \\ 1 \end{bmatrix} \right\}$.

3. Find an orthnormal basis for the space which is spanned by $\left\{ \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ -2 \end{bmatrix} \right\}$ in $\mathbb{R}^2$.

4. The set $B = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} \right\}$ is a basis of $\mathbb{R}^3$. Use the Gram-Schmidt process to create an orthonormal basis of $\mathbb{R}^3$.