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Linear transformation (Linear Map)

$T : V \rightarrow W$ is a f^n s.t.

Vector sp   Vector sp

$\forall \bar{u}, \bar{v} \in V, \forall \alpha, \beta \in \mathbb{R}$

$$T(\alpha \bar{u} + \beta \bar{v}) = \alpha T(\bar{u}) + \beta T(\bar{v})$$

* The set of linear maps is also a vector space

More on this later

Properties of lin. maps

(i) $T(\vec{0}) = \vec{0}$

(ii) $T(-\vec{v}) = -T(\vec{v}) \quad \forall v \in V$

(iii) Lin. property: $T(c_1\bar{v}_1 + c_2\bar{v}_2 + \dots + c_n\bar{v}_n) = c_1T(v_1) + \dots + c_nT(\bar{v}_n)$

Q1) Is $T(x) = 5x \quad \forall x \in \mathbb{R}$, $T : \mathbb{R} \rightarrow \mathbb{R}$, a linear map? Yes!

Q2) Is $T(x) = 5x + 7 \quad \forall x \in \mathbb{R}$, $T : \mathbb{R} \rightarrow \mathbb{R}$, a linear map? Why? No!

It's actually an affine map.

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$T: V \to W$. Let $V = \mathbb{R}^n$, $W = \mathbb{R}^m$

$(e_1, e_2, \ldots, e_n)$ are basis of V

then the matrix rep. of the linear map. $(T(\bar{v}) = A\bar{v},\ \forall \bar{v} \in \mathbb{R}^n)$

$$A = \begin{pmatrix} | & | & & | \\ T(e_1) & T(e_2) & \cdots & T(e_n) \\ | & | & & | \end{pmatrix}$$

where

$e_i = i^{th}$ std (canonical) basis element of $\mathbb{R}^n$.

Q1) Find the matrix representation B of the lin. transf^n $T: P_2 \to P_2$ defined as: $T(f(x)) = f'(x) + f''(x)$. Use the std. basis $= \{1, x, x^2\}$

If you knew no calculus but had access to a dictionary of Bases matrices like B, you could still do calculus using Linear Algebra!

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Ans:—

$$B = \begin{pmatrix} | & | & | \\ T(1) & T(x) & T(x^2) \\ | & | & | \end{pmatrix}$$

$$\begin{cases} T(1) = 0 + 0 = 0 = 0 \times 1 + 0 \times x + 0 \times x^2 \\ T(x) = 1 + 0 = 1 = 1 \times 1 + 0 \times x + 0 \times x^2 \\ T(x^2) = 2x + 2 = 2 \times 1 + 2 \times x + 0 \times x^2 \end{cases}$$

$$\to B = \begin{pmatrix} 0 & 1 & 2 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{pmatrix}$$

Q2) Consider the poly $q(x) = -x + 3$. Compute $T(q)$.

Ans: $Tq = Bq = \begin{pmatrix} 0 & 1 & 2 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 3 \\ -1 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$

$Tq = -1 \times 1 + 0 \times x + 0 \times x^2 = -1$

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