Pay Me My Money Down

Objective of the experiment: To illustrate the concept and application of compound Poisson distribution and to learn to implement the crude Monte-Carlo simulation to estimate probabilities.

Learning concepts: compound Poisson distribution, Monte-Carlo simulation, law of large numbers, insurance payouts.

Pay Me My Money Down is a beautiful work-song that originated among the Negro stevedores working in the Georgia Sea Islands, USA. The song became a sensation among the working millennials due to Bruce Springsteen.[1]

Synopsis

1 Title: *Predicting insurance claim aggregates during a policy period*

1.1 Epilogue: Modeling insurance claims using a compound probability distribution

A certain insurance company is interested in predicting the total aggregate of all claims made during a fixed policy period from a portfolio of insurance products. Such an exercise will enable the company to make an assessment of its financial risks while charting out product launch schedules for the upcoming financial year.

A consultant to the company designs the following mathematical model to accomplish this task. Consider that the firm expects a certain number (N_j) of claims, from amongst its clients, during a fixed period j. Since there is no reason for this number N_j to be deterministically computable,[2] it is reasonable to assume N_j to be a random variable. Now there are N_j of these claims, each claim amount is independent of the other and is also independent of N_j. This is also reasonable because each claim is made by a different client acting independent of the other. Further, each claim amount is also a random number which possibly corresponds to a common probability distribution. Let the claim amount by the i^{th} client be denoted by X_i. X_i corresponds to a probability distribution function $F_X(x)$. The aggregate claim for the policy period j under consideration is also a random quantity $Y_j = X_1 + X_2 + \cdots X_{N_j} = \sum_{i=1}^{N_j} X_i$ that obeys a compound probability distribution. Based on this model, a quantity of interest to the insurance firm is $E(Y_j)$ that you as the consultant will have to estimate in this project.

Moreover, consider there are four policy periods in a given financial year. The total premium collected at the beginning of the year by the insurance firm is $\$\ m$. Let λ_j be the rate at which claims are received per policy period j. Now consider $Z = \sum_{k=1}^{4} Y_k$ is the aggregate claim at the end of the 4^{th} policy period (year end). The company incurs a loss if $Z > \$\ m$. In this project, you will simulate a certain compound stochastic process in Matlab and compute the associated risk for the insurance firm in terms of a probability $P(Z > \$\ m)$. Concurrently, you will learn about a composite stochastic model known as the *compound Poisson process* that is used by insurance companies to assess their risks.

[1] https://en.wikipedia.org/wiki/Pay_Me_My_Money_Down

[2] A multitude of external factors may determine the value of N_j. The complex inter-relationship between these factors may further enhance the uncertainty in knowing what the exact value of N_j might be.

The nuts and bolts

1.2 Interlude: Computing the moments of the compound Poisson distribution and estimating aggregate insurance claims by clients by theoretical analysis

Consider $Y_j = X_1 + X_2 + \cdots + X_{N_j}$ is the aggregate of a random number of claims N_j per quarter (policy period) where $N_j \sim Poisson(\lambda_j),\ j = 1, 2, 3, 4$ (corresponding to each of four quarters) and $X_i \sim Bernoulli([1, 2], p_2)$ are individual claims with probability $p_1 = \frac{2}{3}$ and $p_2 = \frac{1}{3}$ corresponding to claims denominations of \$ $100,000$ and \$ $200,000$ respectively. Further, $\lambda_1 = 2,\ \lambda_2 = 3,\ \lambda_3 = 1,\ \lambda_4 = 3$. $Z = \sum_{j=1}^{4} Y_j$ is the yearly total of all claims made to the firm. Answer the following questions.

1. Identify the distribution of Y_j.
2. Compute $E(Z)$ and $Var(Z)$.
3. Compute $P(Y_2 > 5)$ and compute $P(Y_3 > 5)$ analytically (without a computer simulation). Subsequently, comment on the discrepancy between the two results (if any).

Crank up the Monte-Carlo engine

1.3 Prologue: Predicting risk of monetary loss associated with the insurance scheme for the company using a Monte Carlo simulation

In this section, use the *crude* Monte Carlo simulation (and thereby the law of large numbers) to predict the following.

1. Estimate $P(Y_2 > 5)$ and $P(Y_3 > 5)$ using the crude Monte Carlo simulation. Compare your simulation results here with the analytical results you obtained in section 1.2. Comment on your comparisons.
2. Let the total annual income on the sale of insurance premiums be \$ $1,000,000$. What is the risk of yearly loss for the company in terms of $P(Z > 1,000,000)$? You may provide your analysis of the risk by using an appropriate Monte Carlo simulation.

Pseudo-code for the Monte-Carlo algorithm

The pseudo-code for implementing the Monte-Carlo simulation is available in the laboratory handout here: Rev-up my Monte-Carlo engine

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