Experiment: 6
Gram Schmidt and QR Factorization

1. Gram Schmidt Process

Let V be any vector space and $\{v_1, v_2, \ldots, v_n\} \subseteq V$. Now we want to convert this n-vectors into orthonormal vectors using Gram Schimdt process.

(a) Convert the vectors $\left\{ \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ -2 \end{bmatrix} \right\}$into orthonormal vectors.

(b) Convert the vectors into $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} \right\}$ into orthonormal vectors.

Algorithm

// Output: Matrix B

- Define a matrix A whose columns are $v_1, v_2, \ldots, v_n$ ($A = [v_1\ v_2\ ..\ v_n]$)
- if the rank of A is not equal to n.
  Display the given vectors are linearly dependent
  else
  Define an empty array B in which you want to store the orthonormal vectors
- Set $v =$ first column of A and $u = \frac{v}{||v||}$
- Add u into B as first column.
- for i from 2 to n
  Set v=zeros(n,1), parlv=0, u =zeros(n,1)
  for j form 1 to i-1
  parlv=parlv+$(B(:,j)' * A(:,i)) * B(:,j)$
  end for
  $v = A(:,i) -$ parlv
  $u = \frac{v}{||v||}$
  Add u into B as a column.
  end for
- Display the matrix B.

2. QR factorization using Gram–Schmidt process

Implement this algorithm as MATLAB function script.

(a) Find the QR factorization of following matrix

$$\begin{bmatrix} 1 & -1 & 1 \\ -2 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$$

Algorithm

// Input: Matrix A

// Output: Matrix Q and R.

- Create a function which take the matrix A as input
- Define dimension of matrix A(say $m \times n$)
- if rank of A is not equal to n
  Factorization does not exists
  return to main script
  else
  Define an empty array Q
  and a null matrix R of size $n \times n$
- Set $v =$ first column of A and $u = \frac{v}{||v||}$
- Add u into Q as first column.
- for i from 2 to n
  Set v=zeros(n,1), parlv=0, u =zeros(n,1)
  for j form 1 to i-1
  parlv=parlv+$(Q(:,j)' * A(:,i)) * Q(:,j)$
  end for
  $v = A(:,i) -$ parlv
  $u = \frac{v}{||v||}$
  Add u into Q as a column.
  end for
- for i from 1 to n
  for j from i to n
  $R_{ij} =< Q(:,i), A(:,j) >$
  end for
  end for