Computational Linear Algebra: FM 126 — Total time: 1 hour

Quiz-1 for section: L2 — February 13, 2025

Full Name: ______________________ **UID:** ______________________

Instructions: You must **not** be in possession of any cheat sheet, notes, or electronic devices like laptops or calculators inside the examination hall. Answer **all ten multiple-choice questions (MCQs)**. The score allotted to each question is **one**. There will be a penalty of **0.25 marks** for each wrong answer. ***If you mark more than one option as your answer to any question, your response will be treated as incorrect and the penalty will apply (even if one of the opted answers is the correct answer).*** Darken the circle against the correct option. **Maximum score is 10**.

========================START OF QUESTIONS========================

1. Consider $V = \left\{ \begin{bmatrix} a & b \\ 1 & 3 \end{bmatrix} \text{ such that } a, b \in \mathbb{R} \right\}$ with vector addition defined as

$$\begin{bmatrix} a_1 & b_1 \\ 1 & 3 \end{bmatrix} + \begin{bmatrix} a_2 & b_2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} a_1 + a_2 & b_1 + b_2 \\ 1 & 3 \end{bmatrix}; \quad \text{for all} \quad a_1, b_1, a_2, b_2 \in \mathbb{R},$$

and scalar multiplication defined as

$$k \begin{bmatrix} a & b \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} ka & kb \\ 1 & 3 \end{bmatrix}; \quad \text{for all} \quad k, a, b \in \mathbb{R}.$$

Which of the following is true?

- ◯ V is not a vector space because V is not closed under the given vector addition.
- √ *V is a vector space with given vector addition and scalar multiplication.*
- ◯ V is not a vector space because V is not closed under the given scalar multiplication.
- ◯ V is not a vector space because there is no zero vector in V.

2. Consider the vector space $V = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \text{ such that } x = y + z, y = z \quad \text{and } x, y, z \in \mathbb{R} \right\}$ with usual vector addition and scalar multiplication. Which of the following is true?

√ *$dim(V) = 1$* ◯ $dim(V) = 2$ ◯ $dim(V) = 3$ ◯ $\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \right\}$ is a basis of V.

3. Let V be a finite dimensional vector space of dimension $\mathbf{n}$ defined over the field $\mathbb{R}$. Which of the following statement is **NOT** true?

- ◯ Any spanning set S of V with exactly $\mathbf{n+1}$ vectors is a linearly dependent subset of V.
- √ *Any spanning set S of V with exactly $\mathbf{n+1}$ vectors is a linearly independent subset of V.*
- ◯ Any spanning set S of V with exactly $\mathbf{n}$ vectors forms a basis of V.
- ◯ Any spanning set S of V with exactly $\mathbf{n}$ vectors is a linearly independent subset of V.

4. Let $S = \left\{ \begin{pmatrix} 1 \\ \alpha \end{pmatrix}, \begin{pmatrix} \alpha \\ 1 \end{pmatrix} \text{ such that } \alpha \in \mathbb{R} \right\}$. Then S is a basis for vector space $\mathbb{R}^2$ under usual vector addition and scalar multiplication of vectors if and only if

√ *$\alpha^2 \neq 1$* ◯ $\alpha \neq 1$ ◯ $\alpha^2 = 1$ ◯ $\alpha \neq -1$

5. For what value of m, the vector $\begin{pmatrix} m \\ 3 \\ 1 \end{pmatrix}$ is a linear combination of the vectors $\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$ under usual vector addition and scalar multiplication of vectors?

◯ 1 √ *5* ◯ 2 ◯ 3

6. Consider a vector space $V = \{a_0 + a_1 x + a_2 x^2 + a_3 x^3 \text{ such that } a_0, a_1, a_2, a_3 \in \mathbb{R}\}$ under usual vector addition and scalar multiplication of polynomials. Which one of the following statements is correct?

- ◯ dim(V)=2
- ◯ dim(V)=3
- √ *$\{1, x, x^2, x^3\}$ is a basis*

7. Consider the vector space $V = \{(x, y, z) \in \mathbb{R}^3$ such that $y = x\}$ of $\mathbb{R}^3$. The given vector space is spanned by which one of the following set?

○ $\left\{\begin{pmatrix}1\\1\\0\end{pmatrix}, \begin{pmatrix}\pi\\\pi\\0\end{pmatrix}\right\}$

○ $\left\{\begin{pmatrix}1\\0\\0\end{pmatrix}, \begin{pmatrix}0\\1\\0\end{pmatrix}\right\}$

√ $\left\{\begin{pmatrix}1\\1\\0\end{pmatrix}, \begin{pmatrix}-2\\-2\\0\end{pmatrix}, \begin{pmatrix}0\\0\\\pi\end{pmatrix}\right\}$

○ $\left\{\begin{pmatrix}1\\0\\0\end{pmatrix}, \begin{pmatrix}0\\0\\1\end{pmatrix}\right\}$

8. What is the rank of matrix $A = \begin{bmatrix}5 & 5 & 6 & 7\\2 & 2 & 3 & 3\\4 & 4 & 5 & 6\end{bmatrix}$?

○ 0 ○ 1 ○ 2 √ *3*

9. For the matrix $B = \begin{bmatrix}1 & 2 & 0\\0 & 1 & 1\\1 & 3 & 1\end{bmatrix}$. Which one of the following is **NOT** the basis of the column space of the matrix B?

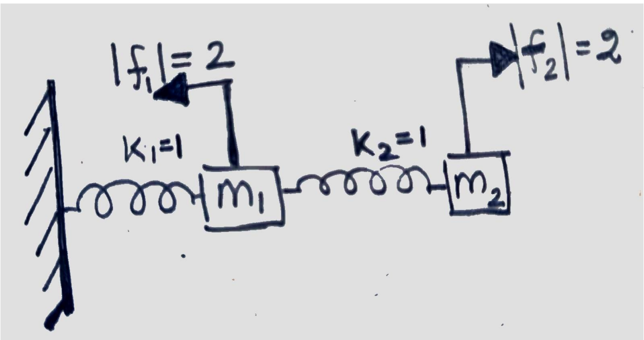
○ $\left\{\begin{bmatrix}1\\0\\1\end{bmatrix}, \begin{bmatrix}2\\1\\3\end{bmatrix}\right\}$

○ $\left\{\begin{bmatrix}2\\1\\3\end{bmatrix}, \begin{bmatrix}0\\1\\1\end{bmatrix}\right\}$

○ $\left\{\begin{bmatrix}1\\0\\1\end{bmatrix}, \begin{bmatrix}0\\1\\1\end{bmatrix}\right\}$

√ $\left\{\begin{bmatrix}1\\0\\1\end{bmatrix}, \begin{bmatrix}2\\1\\3\end{bmatrix}, \begin{bmatrix}0\\1\\1\end{bmatrix}\right\}$

10. Consider the following spring-mass system with spring constants $k_1 = k_2 = 1$. Opposing forces of equal magnitude are applied to the masses as shown in the picture. Write the linear system model for this situation in terms of $\mathbb{A}\mathbf{u} = \mathbf{f}$. Which of the following is true?



○ The system has one unique equilibrium solution that depends on the exact values of both k_1 and k_2.

○ The system has no equilibrium solutions.

○ None of these.

○ The system has a family of solutions that depends on the exact given values of both k_1 and k_2 ($k_1 = k_2 = 1$).

√ *The system has one unique equilibrium solution that does not depend on the value of the spring constant k_1.*