Engineering Maths in Action: FM 103 — Total time: 30 mins

Quiz-1 for section-L1 — November 02, 2023

Full Name: ______________________ **UID:** ______________________

Instructions: You must **not** be in possession of any cheat sheet, notes, or electronic devices like laptops or calculators inside the examination hall. Answer **all five multiple-choice questions (MCQs)**. The score allotted to each question is **one**. There will be a penalty of **0.5 marks** for each wrong answer and a penalty of **0.25 marks** for each un-attempted question. **Maximum score is 5**. Tick against the correct option. Only one option is correct in every question.

========================START OF QUESTIONS========================

1. Let A be a null matrix of order 3×3, then the eigenspace (the set of all eigenvectors of A) corresponding to the eigenvalues of matrix A is

   ○ $\begin{bmatrix}0\\0\\0\end{bmatrix}$ ○ $\begin{bmatrix}0\\0\\0\end{bmatrix}, \begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}$ ○ $\begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix}$ ○ None of these

2. Let V be the subspace of $\mathbb{R}^4$ spanned by $u = \{1, 1, 1, 1\}$ and $v = \{1, 9, 9, 1\}$. The orthonormal bases of V obtained by the Gram-Schmidt orthonormalization process are:

   ○ $\left\{\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right), \left(-\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}\right)\right\}$
   ○ $\{(1, 1, 0, 0), (1, 0, 1, 0)\}$
   ○ $\left\{\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right), \left(\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}\right)\right\}$
   ○ $\left\{\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right), \left(\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}\right)\right\}$

3. Let T be a linear transformation which is a projection of the space $\mathbb{R}^3$ to the xy-plane embedded in $\mathbb{R}^3$. What are the eigenvalues of the matrix representation of T?

   ○ 0,0,1 ○ 0,1,1 ○ 1,1,1, ○ 0,0,0

4. Find the matrix A^3, if matrix the A is diagonalizable such that $D = S^{-1}AS$, where matrix $S = \begin{bmatrix}2 & -1\\5 & 1\end{bmatrix}$ and matrix $D = \begin{bmatrix}6 & 0\\0 & -1\end{bmatrix}$?

   ○ $\begin{bmatrix}61 & 62\\156 & 154\end{bmatrix}$
   ○ $\begin{bmatrix}8 & -1\\125 & 1\end{bmatrix}$
   ○ $\begin{bmatrix}61 & 62\\155 & 154\end{bmatrix}$
   ○ $\begin{bmatrix}216 & 0\\0 & -1\end{bmatrix}$

5. The set of linearly independent eigenvectors corresponding to the eigenvalues of the matrix $\begin{bmatrix}2 & -1\\2 & 4\end{bmatrix}$ is:

   ○ $\{(i, 1), (1, i)\}$
   ○ $\{(i, -1), (-i, -1)\}$
   ○ $\{(1, -1 - i), (1, -1 + i)\}$
   ○ None of these