PLAKSHA UNIVERSITY

Computational Linear Algebra: FM 126 | Total time: 1 hour

Quiz-1 for section: L1 | February 11, 2025

Full Name: ____________________ **UID:** ____________________

Instructions: You must **not** be in possession of any cheat sheet, notes, or electronic devices like laptops or calculators inside the examination hall. Answer **all ten multiple-choice questions (MCQs)**. The score allotted to each question is **one**. There will be a penalty of **0.25 marks** for each wrong answer. ***If you mark more than one option as your answer to any question, your response will be treated as incorrect and the penalty will apply (even if one of the opted answers is the correct answer).*** Darken the circle against the correct option. **Maximum score is 10**.

========================START OF QUESTIONS========================

1. Consider $V = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \text{ such that } a, b, c, d \in \mathbb{R} \right\}$. V is a vector space under usual matrix addition and scalar multiplication.
   Consider the following linear combination of vectors in V.
   $$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = a \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + b \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \text{for all } \ a = d \text{ and } b = c. \tag{i}$$
   Which of the following is true?
   - √ *None of these.*
   - ○ $\left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \right\}$ spans V.
   - ○ Considering the above linear combination of basis vectors $\left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \right\}$ in (i), it may be concluded that $dim(V) = 2$.
   - ○ $\left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \right\}$ is a basis of V.

2. Statement 1 : $\left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} -5 \\ 4 \end{pmatrix}, \begin{pmatrix} 3 \\ 1 \end{pmatrix} \right\}$ is a linearly independent subset of $\mathbb{R}^2$.
   Statement 2 : $\left\{ \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} -5 \\ 4 \\ 2 \end{pmatrix} \right\}$ is a linearly independent subset set of $\mathbb{R}^3$
   Then:
   - ○ Both are true.
   - ○ Statement 1 is true, but Statement 2 is false.
   - ○ Both are false.
   - √ *Statement 2 is true, but Statement 1 is false.*

3. Consider $V = \{ax^2 + bx + c \text{ such that } a, b, c \in \mathbb{R}\}$ with the usual addition and scalar multiplication of polynomials. Which of the following is true?
   - ○ If $c = 1$, then V is a vector space with dimension 2.
   - √ *If $c = 0$, then V is a vector space with dimension 2.*
   - ○ If $c = 1$, then V is a vector space with dimension 3.
   - ○ If $c = 0$, then V is a vector space with dimension 3.

4. If $S = \{v_1, v_2, v_3\}$ is a basis for a vector space V. For any v_4 in V, which of the following statement is correct?
   - ○ $S \cup \{v_4\}$ is always linearly independent.
   - ○ $S \cup \{v_4\}$ is a basis only if v_4 is not a linear combination of v_1, v_2, v_3.
   - √ *$S \cup \{v_4\}$ is always linearly dependent.*
   - ○ $S \cup \{v_4\}$ is still a basis.

5. Let $S = \left\{ \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \right\}$. The value of k for which the vector $\begin{pmatrix} 2 \\ 1 \\ k \end{pmatrix}$ belongs to the span of S, is
   - ◯ 2
   - √ *0*
   - ◯ -1
   - ◯ 1

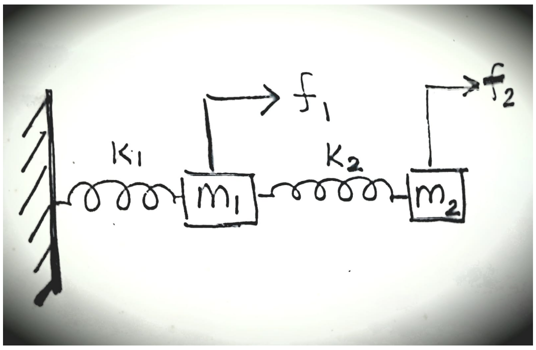
6. Dimension of vector space $W = \left\{ \begin{pmatrix} a \\ b \\ c \end{pmatrix} \text{ such that } a + b = c \right\}$ over $\mathbb{R}$ is equal to
   - √ *2*
   - ◯ 1
   - ◯ None of these.
   - ◯ 3

7. Let V be the vector space of all 5×5 matrices over the field $\mathbb{R}$. Then the dimension of V is
   - ◯ -10
   - √ *25*
   - ◯ 10
   - ◯ 0

8. Consider $S = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \text{ such that } x, y, z \in \mathbb{R} \text{ and } z \neq 0 \right\}$
   - ◯ S is a vector space.
   - √ *S is not a vector space.*
   - ◯ None of these.

9. If B and $B^{'}$ are two different bases of the vector space V, Then
   - ◯ B and $B^{'}$ have different number of elements.
   - √ *B and $B^{'}$ have same number of elements.*
   - ◯ None of these.
   - ◯ $B = B^{'}$.

10. Consider the following spring-mass system with spring constants $k_1 = k_2 = 1$. Slowly, apply the steady forces, $f_1 = 1$ and $f_2 = 0$, as shown in the figure below. At the instance of applying these forces, we slowly cut the second spring (effectively resetting $k_2 = 0$). Consequently, the displacements of the respective masses are u_1 and u_2 upon the application of the forces. Which of the following is true?



   - √ *The system has one unique equilibrium solution with $u_2 = 0$ and $u_1 = 1$.*
   - ◯ The system has no equilibrium solution.
   - ◯ The system has one unique equilibrium solution because $\mathbf{f} = \begin{pmatrix} f_1 \\ f_2 \end{pmatrix}$ is the resultant vector of the vectors $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$.
   - ◯ The system has infinitely many equilibrium solutions because $\mathbf{f} = \begin{pmatrix} f_1 \\ f_2 \end{pmatrix}$ is in the span of the column vectors of the stiffness matrix $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$.