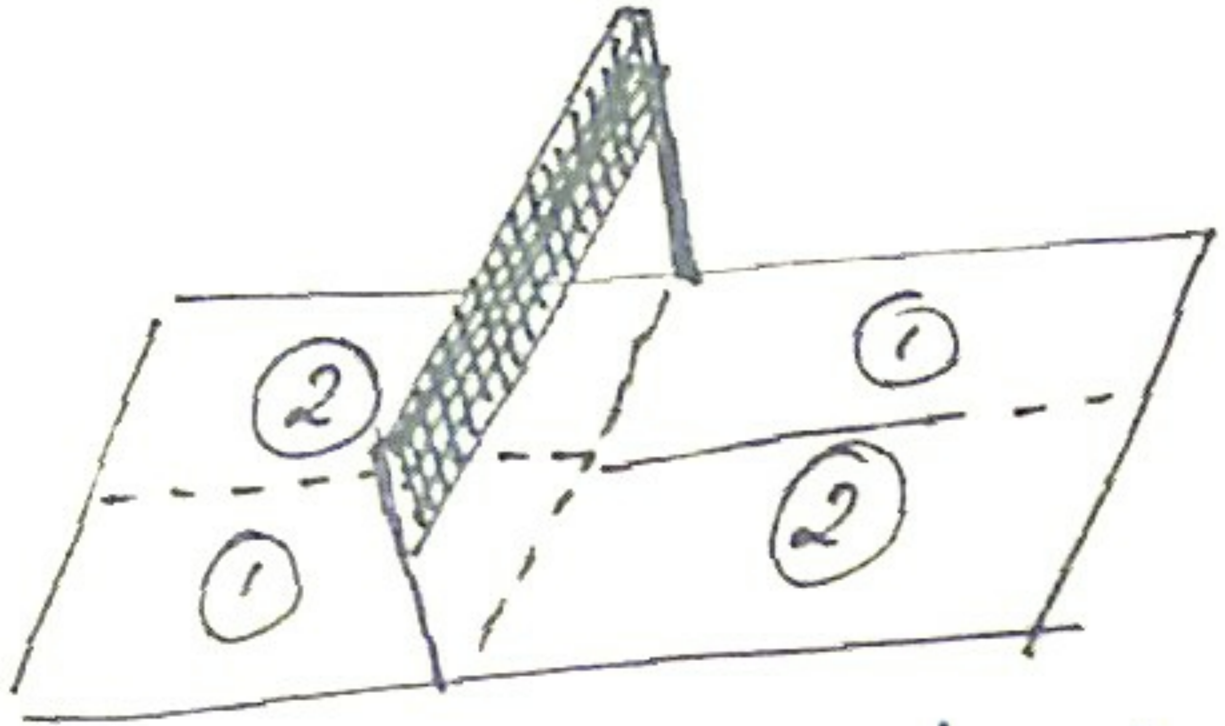
Numerical problems on return times.



Q) Consider a simple badminton court divided into 2 quadrants on each side as shown.
Based on recorded scores/rallies over many games in the past, the data reveals that the probability "a cross-court shot is played" is $2/3$. Assume a Markov model of the game.

i) Construct the probability transition matrix.

ii) Given a service from quadrant 1; when does the shuttle return to quadrant 1 on an average on either side?

Soln :- $\mu_1(1) = 1 + p_{12}\,\mu_2(1)$

$$= 1 + \frac{1}{3}\mu_2(1)$$

$$\mu_2(1) = 1 + p_{22}\,\mu_2(1)$$

$$= 1 + \frac{2}{3}\mu_2(1)$$

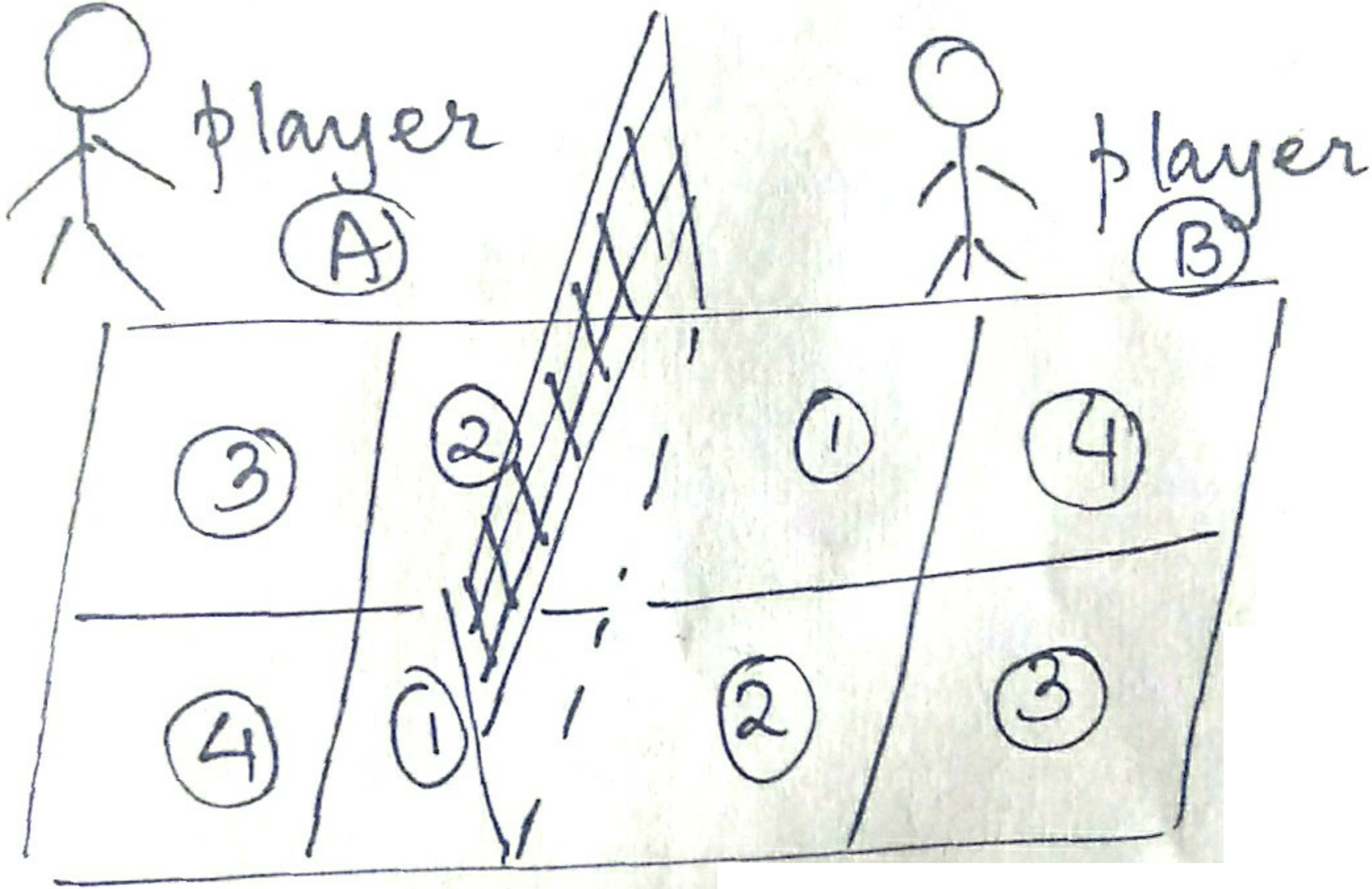
Recall formula from part III of previous lecture on mean return time to state y starting at state x

$$\boxed{\mu_x(y) = 1 + \sum_{\substack{m \in S \\ m \neq y}} p_{xm}\,\mu_m(y)}$$

$$\Rightarrow \frac{1}{3}\mu_2(1) = 1 \Rightarrow \mu_2(1) = 3$$

$$\therefore \mu_1(1) = 1 + \frac{1}{3} \times 3 = 2$$

i.e. every 2nd shot in the rally returns to quadrant 1 given a service from quadrant 1.



player
A
player
B
3
2
1
4
4
1
2
3

Q) Now consider an extended (& more realistic) version of the same problem where the badminton court is divided into 4 quadrants. At any given instant in a rally, the shuttle arrives in one of these 4 quadrants according to a Markov process:-

Quadrant (n^{th} shot)	Quadrant ($n+1^{th}$ shot)	w/ prob.
1	1	0
	2	1/4
	3	1/4
	4	1/2
2	1	1/2
	2	0
	3	1/4
	4	1/4
3	1	1/4
	2	0
	3	1/4
	4	1/2

Quadrant (n^{th} shot)	Quadrant ($n+1^{th}$ shot)	w/ prob.
4	1	0
	2	1/4
	3	1/2
	4	1/4

Q1) Construct the probability transition matrix

Q2) Given that in an instant of a game, player A serves from quadrant 1; what is the average no. of shots in that rally before the shuttle arrives again in quadrant 1 for either of the two players.