Questions

1. Each of the following real matrices defines a linear transformation on $\mathbb{R}^2$:

   (a) $A = \begin{bmatrix} 5 & 6 \\ 3 & -2 \end{bmatrix}$

   (b) $B = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$

   (c) $C = \begin{bmatrix} 5 & -1 \\ 1 & 3 \end{bmatrix}$

   Find, for each matrix, all eigenvalues and a maximum set of linearly independent eigenvectors. Which of these linear operators are diagonalizable—that is, which can be represented by a diagonal matrix?

2. Suppose the matrix B in Problem 1 represents a linear operator on complex space $\mathbb{C}^2$. Show that, in this case, B is diagonalizable by finding a basis S of $\mathbb{C}^2$ consisting of eigenvectors of B.

3. Determine whether the matrix:

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 2 & 4 \end{bmatrix}.$$

   is diagonalizable. If yes, find a suitable matrix S such that $A = S^{-1}DS$.

4. Let

$$A = \begin{bmatrix} 3 & -1 & 1 \\ 7 & -5 & 1 \\ 6 & -6 & 2 \end{bmatrix}.$$

   Find its eigenvalues and eigenvectors. Check whether A is diagonalizable.

5. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be defined by:

$$T(x, y, z) = (2x + y - 2z, 2x + 3y - 4z, x + y - z).$$

   (a) Find all eigenvalues of T.

   (b) Find a basis of each eigenspace.

   (c) Is T diagonalizable? If so, find the basis S of $\mathbb{R}^3$ that diagonalizes T and find its diagonal representation D.