PLAKSHA UNIVERSITY

PLAKSHA UNIVERSITY, MONSOON SEMESTER AY 2024-25

Course Code: FM 216

Course Title: Computational Methods and Optimization

Course Credits: 03 **L / T / P: 2/0/1**

Course intended for: 3rd semester undergraduate students

Prerequisites: Basic knowledge of calculus, familiarity with linear algebra and differential equations, introductory skills in the PYTHON programming language

Class Schedule: L1: T Th 10:00 AM – 10:50 AM, L2: T Th 09:00 AM – 09:50 AM

Classroom No.: 2201, 1002

Lab Schedule: L1P1: W 02:00 PM – 03:50 PM, L1P2: Th 02:00 PM – 03:50 PM, L2P1: M 02:00 PM – 03:50 PM, L2P2: T 02:00 PM – 03:50 PM

Lab room number: L1P1: 2101, L1P2: 2101, L2P1: 2101, L2P2: 2101

Lead Instructor: Amrik Sen, Associate Professor

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Co-Instructor(s): None

Office No. NA

Email ID: NA

Teaching Fellows: Mr. Rajat Singla, Mr. Vijay Sahani, Mr. Viraj Dsouza

Course Description

This course provides an introduction to computational methods for solving optimization problems. Students will be introduced to vector calculus as a pre-requisite to the module on optimization techniques. We will discuss several applications from operations research and applied engineering that rely on linear programming methods and constrained optimization techniques. Laboratory work will train and test students' comprehension through live projects.

Course Overview

The course is divided into four parts, namely,

1. **Numerical methods:** This will include root finding techniques that have applications in a very important class of optimization methods known as gradient descent algorithms. Secondly, we will discuss numerical strategies to compute integrals of complex functions.

2. **Vector Calculus:** This section will include analysis of multivariate functions, computing higher order derivatives, finding extrema of multivariate functions, and higher order Taylor expansion of multi-variate functions. We will also study the concept of gradient of a scalar function, divergence and curl of a vector field and their physical significance.

3. **Constrained and unconstrained optimization based on vector calculus:** This section will include the notion of a Hessian and application of Lagrange multipliers to solve problems in optimization.

4. **Optimization paradigms:**
    a. **Linear Programming Principles:** In this section we will study a class of engineering problems that involve finding optimal solutions of linear objective functions subject to linear equality and linear inequality constraints. The region of feasibility is a convex polytope.
    b. **Dynamic Programming Principles:** In this section we will introduce students to an alternative yet powerful optimization paradigm that involves decomposing complex decision making objectives into simpler and smaller decision steps iteratively. Specifically, we will discuss the *travelling salesman* problem as a case-study application example.

Learning Objectives

1. The course will familiarize students with various optimization techniques including unconstrained and constrained optimization of functions.
2. The course will train students to apply key concepts of vector calculus — such as gradients, divergence, and the Hessian matrix — which are essential to analyze and solve optimization problems.
3. The course will impart students with the skills to implement optimization algorithms using Python.
4. The course will enable students to critically analyze the performance and effectiveness of various optimization methods.
5. The course will introduce students to the exciting world of operations research and mathematical modeling through applied engineering projects.

Learning Outcomes:

By the end of the course, students will be able to:

1. compute integration of complex mathematical functions using numerical techniques,
2. compute roots of mathematical functions using computational methods,
3. analyze vector valued functions – find extrema, saddle points, etc.
4. compute curl and divergence of vector fields,
5. solve problems in operations research using optimization techniques such as linear programming methods, dynamic programming, etc.

Recommended Textbooks

I. **Numerical Analysis** by *Richard L. Burden and J. Douglas Faires*, 9th edition, Cengage Learning, Inc, (2010).

II. **Thomas' Calculus** by *J. Hass, C. Heil and M. D. Weir*, 15th edition, Pearson Education, (2024).

III. **Calculus** by *H. Anton, I. Bivens and S. Davis*, 10th edition, John Wiley & Sons, Singapore, (2016).

IV. **Introduction to Operations Research** by *Frederick S. Hillier and Gerald J. Lieberman*, 10th edition, McGraw-Hill Education, (2014).

Assessments and Grading:

- **Term Exams: 50%**
  - Mid-semester exam (20%) Date: October 03–09, 2024 (Week-8, 9)
  - Final comprehensive exam (30%) Date: December 13–19, 2024 (Week-18,19)
- **Project: 30%**
  - Part I – 10% Date: Week-5, 6
  - Part II – 20% Date: Week-14, 15, 16
- **Attendance: 10%** (mandatory for all UG courses)

  >80% = **10 points**

  70 to 80 % = **8 points**

  60 to 70 % = **6 points**

  < 60 % = **0 points**
- **Quiz: 10%**
  - Quiz 1: October 26, 2024 (Week–11)
  - Quiz 2: December 07, 2024 (Week–17)

Weekly Class Plan

Date	**Topics**	**Readings** **(To be done prior to class meeting)**	**Assignments** **Due date**
Week 1 (August 12-16, 2024)	Introduction 1 (syllabus) Lab: Plotting of functions		
Week 2 (August 19-23, 2024)	Introduction 2 (numerical optimization) Root-finding problems, bisection method Lab: Taylor series of functions and convergence	Textbook: Burden and Faires (Chapter-2)	

Week 3 (August 26-30, 2024)	Newton-Raphson method Secant method, error analysis Lab: Implementation of bisection method	Textbook: Burden and Faires (Chapter-2)	
Week 4 (September 02-06, 2024)	Fixed-point iteration Root finding methods (summary & conclusion) Lab: Root-finding problems using numerical methods and error analyses	Textbook: Burden and Faires (Chapter-2)	
Week 5 (September 09-13, 2024)	Numerical integration, trapezoidal rule Simpson's rule Lab: Project	Textbook: Burden and Faires (Chapter-4)	Project part 1, interview–1A
Week 6 (September 16-20, 2024)	Error analysis for numerical integration Numerical integration (summary & conclusion) Lab: Project	Textbook: Burden and Faires (Chapter-4)	Project part 1, interview–1B
Week 7 (September 23-27, 2024)	Multivariate functions, partial derivatives. Tangent vector and tangent plane Lab: Implementation of the trapezoidal rule and Simpson's rule to approximate integrals and error analyses	Textbook: Thomas' Calculus (Chapter–13)	

Week 8 (September 30 - October 04, 2024)	Revision of vector calculus Mid-semester exam Lab: Plotting multivariate functions	Textbook: Thomas' Calculus (Chapter–13)	Mid–sem exam
Week 9 (October 07-11, 2024)	Mid-semester exam Higher-order partial derivatives, mixed partial derivatives, total derivative Lab: Plotting multivariate functions.	Textbook: Thomas' Calculus (Chapter–13)	Mid–sem exam
Week 10 (October 14-18, 2024)	Directional derivatives, gradients, curl and divergence Taylor series for single and multi-variate functions Lab: Higher order derivatives of multivariate functions	Textbook: Thomas' Calculus (Chapter–13)	
Week 11 (October 21-25, 2024)	Extreme values, saddle points, first derivative tests for local extrema Second derivative test for local extrema (Hessian) Lab: Gradients, curl, divergence, and Taylor series expansion of multivariate functions	Textbook: Thomas' Calculus (Chapter–13)	Quiz 1: October 26, 2024
Week 12 (October 28 – November 01, 2024)	Examples: extrema of multivariable functions Lab: Implement methods to find local extrema and saddle points of multivariable functions	Textbook: Thomas' Calculus (Chapter–13)	

Week 13 (November 04-08, 2024)	Lagrange multipliers for constrained optimization - 1 Lagrange multipliers for constrained optimization – 2 Lab: Implement Lagrange multipliers to solve constrained optimization problems	Textbook: Thomas' Calculus (Chapter–13)	
Week 14 (November 11-15, 2024)	Gradient descent for unconstrained optimization - 1 Gradient descent for unconstrained optimization – 2 Lab: Project	Notes	Project part 2, interview–2A
Week 15 (November 18-22, 2024)	Linear programming – 1 (constrained optimization) Linear programming – 2 (constrained optimization) Lab: Project	Textbook: Hillier and Lieberman (Chapter–3)	Project part 2, interview–2B
Week 16 (November 25-29, 2024)	Linear programming – 3 (constrained optimization) Integer programming (constrained optimization) – 1 Lab: Project	Textbook: Hillier and Lieberman (Chapter–3) Textbook: Hillier and Lieberman (Chapter–12)	Project part 2, interview–2C
Week 17 (December 02-06, 2024)	Integer programming – 2	Textbook: Hillier and Lieberman (Chapter–12)	Project (residual interviews)

Week 17 (December 02-06, 2024)	Dynamic programming - 1 (Travelling salesman problem) Lab: Project	Textbook: Hillier and Lieberman (Chapter–11)	Quiz 2: December 07, 2024
Week 18 (December 09-12, 2024)	Dynamic programming - 2 Course recap and summary Exam week (no new lab classes)	Textbook: Hillier and Lieberman (Chapter–11)	End - sem exam

PLAKSHA ACADEMIC POLICY (WILL BE PROVIDED BY OAA)

Makeup Policy

Attendance Policy

Policy on Incompletes

Scholastic Misconduct

Insert University Policy here (will be provided by academic office)

Graphical Abstract (kindly insert a graphical abstract of your course here)