Engineering Maths in Action: FM 103 Total time: 30 mins
Quiz-1 for section-L2 November 02, 2023

Full Name: ____________________ **UID:** ____________________

Instructions: You must **not** be in possession of any cheat sheet, notes, or electronic devices like laptops or calculators inside the examination hall. Answer **all five multiple-choice questions (MCQs)**. The score allotted to each question is **one**. There will be a penalty of **0.5 marks** for each wrong answer and a penalty of **0.25 marks** for each un-attempted question. **Maximum score is 5**. Tick against the correct option. Only one option is correct in every question.

========================START OF QUESTIONS========================

1. Let T be a linear transformation which is a projection of the space $\mathbb{R}^3$ to the x-axis embedded in $\mathbb{R}^3$. What are the eigenvalues of the matrix representation of T?

   ○ 0,0,1 ○ 0,1,1 ○ 1,1,1, ○ 0,0,0

2. Find the matrix S such that the given matrix $M = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$ is diagonalizable ($D = S^{-1}MS$), where D is the diagonal matrix equivalent to matrix M.

   ○ $S = \begin{bmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$

   ○ $S = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

   ○ $S = \begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

   ○ there doesn't exist any S.

3. Let V be the subspace of $\mathbb{R}^3$ spanned by $u = \{1, 1, 1\}$ and $v = \{1, 1, -1\}$. The orthonormal bases of V obtained by the Gram-Schmidt orthonormalization process are:

   ○ $\left\{ \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right), \left(\frac{2}{3}, \frac{2}{3}, \frac{4}{3} \right) \right\}$

   ○ $\{(1, 1, 0), (1, 0, 1)\}$

   ○ $\left\{ \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right), \left(\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}} \right) \right\}$

   ○ $\left\{ \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right), \left(\frac{2}{\sqrt{6}}, -\frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}} \right) \right\}$

4. The set of linearly independent eigenvectors corresponding to the eigenvalues of the matrix $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ is:

   ○ $\{(i, 1), (1, i)\}$

   ○ $\{(i, -1), (-i, -1)\}$

   ○ $\{(i, -1), (1, i)\}$

   ○ None of these

5. Suppose the matrix $A = \begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix}$ has an eigenvalue 1 with associated eigenvector $v = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$. Find $A^5 v$.

   ○ Data is insufficient

   ○ $\begin{bmatrix} \alpha^{50} & \beta^{50} \\ \gamma^{50} & \delta^{50} \end{bmatrix}$

   ○ $\begin{bmatrix} 32 \\ 243 \end{bmatrix}$

   ○ $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$