Example based on Bayes' thm.

Q) The entire o/p of a factory is produced on 3 machines (m/cs) accounting for 20%, 30% and 50% of the factory o/p. The fraction of defective items produced is 5% for 1st m/c, 3% for 2nd m/c and 1% for 3rd m/c. If an item is chosen at random from the total o/p and is ~~formed~~ found to be defective, what is the probability that it was produced by the 3rd m/c?

Soln:-

A_i = event that a randomly chosen item is made by i^{th} m/c $(i = 1, 2, 3)$

B = event that a randomly chosen item is defective.

$P(A_1) = 0.2$, $P(A_2) = 0.3$, $P(A_3) = 0.5$

$P(B|A_1) = 0.05$, $P(B|A_2) = 0.03$, $P(B|A_3) = 0.01$

$P(A_3|B) = ?$

$$P(A_3|B) = \frac{P(B|A_3)P(A_3)}{P(B)} = \frac{(0.01)(0.5)}{P(B|A_1)P(A_1) + P(B|A_2)P(A_2) + P(B|A_3)P(A_3)}$$

$$= \frac{(0.01)(0.5)}{(0.05)(0.2) + (0.03)(0.3) + (0.01)(0.5)} = \frac{5}{24}$$

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