Lecture (5) contd... 29/1/2019

More examples & applications of Cauchy-Riemann eqns.

Q) For each of the following, check the CR eqns. If they are satisfied, find $f'(z)$.

(I) $f(x,y) = x - iy + 1$

Soln :- $f = u + iv$ where $u(x,y) = x+1$ & $v(x,y) = -y$

$\because u_x = 1 \neq v_y = -1$, $u_y = 0 = -v_x$

$\Rightarrow$ CR eqns are not satisfied.

$\Rightarrow f'(z)$ DNE — does not exist!

(II) $f(x,y) = y^3 - 3x^2y + i(x^3 - 3xy^2 + 2)$

$u(x,y) = y^3 - 3x^2y$ and $v(x,y) = x^3 - 3xy^2 + 2$

$\because u_x = -6xy = v_y$; $u_y = 3y^2 - 3x^2 = -v_x$

$\Rightarrow$ CR eqns. are satisfied.

And by def^n we have $f'(z) = \frac{df}{dz} = u_x + iv_x = -iu_y + v_y$

$= 3i(x+iy)^2$

$= 3iz^2$

b/c $f(z) = iz^3 + 2i = f(x,y)$.

(III) $f(x,y) = e^y(\cos x + i\sin y) = u + iv$

We have $u_x = -e^y \sin x \neq v_y = e^y \sin y + e^y \cos y$

And $u_y = e^y \cos x \neq -v_x = 0$

$\Rightarrow$ CR eqns. are not satisfied

$\Rightarrow f'(z)$ DNE!

(IV) $f(r,\theta) = \log r + i\theta = u + iv$

$\because u_r = \frac{1}{r} = \frac{v_\theta}{r}$ and $v_r = 0 = -\frac{u_\theta}{r}$ $\Rightarrow$ CR eqns hold

$\Rightarrow f'(z) = e^{-i\theta}(u_r + iv_r) = \frac{e^{-i\theta}}{r} = \frac{1}{z}$.