Similarity Transformation 24/3/25

$D = S^{-1}AS \Rightarrow DS^{-1} = S^{-1}A$.

$A\vec{v} = \lambda\vec{v}$ (λ is the ev of A)

$S^{-1}A\vec{v} = \lambda S^{-1}\vec{v}$

$(DS^{-1})\vec{v} = \lambda S^{-1}\vec{v}$

$\Rightarrow D(S^{-1}\vec{v}) = \lambda(S^{-1}\vec{v})$.

$\Rightarrow D\vec{w} = \lambda\vec{w}$ (λ is the ev of D and $\vec{w}$ is the EV of D

So, A and $D = S^{-1}AS$ have the same evs!

Q) When is a matrix $A \in M_{n\times n}(\mathbb{F})$ diagonalizable?

Ans) When A has n linearly independent EVs

** How do I construct S?

$$S = \begin{pmatrix} | & | & & | \\ v_1 & v_2 & \cdots & v_n \\ | & | & & | \end{pmatrix}$$

where $\{v_i\}, i = 1, 2, \dots, n$ are the n-lin. indep. EVs of A

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Q) So, if in order to find D using S; we have to apriori know the EVs (& hence the eVs) of the original matrix A; what then is the significance of the similarity transformation

$$D = S^{-1} A S \ ?$$

Ans) If we ~~have~~ want to compute A^{55}; we can do so in the following manner:

$$S D S^{-1} = A$$

$$\therefore A^{55} = S D^{55} S^{-1}$$

D^{55} is simply $\begin{pmatrix} \lambda_1^{55} & 0 & 0 & \cdots & 0 \\ 0 & \lambda_2^{55} & 0 & \cdots & 0 \\ \vdots & \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & & \lambda_n^{55} \end{pmatrix}$

✱ ✱ just raise the eVs to the appropriate power!