<u>Syllabus</u>

PMC 103: Complex Analysis

Semester: Spring 2019
Course instructor: Amrik Sen

L	T	P	Credit
3	1	0	3.5

Course Objective: The course aims to introduce the theory of complex analysis to graduate students with applications to solve problems in the mathematical sciences and engineering.

==

Introduction: introduction to complex numbers, geometrical interpretation, different representations of complex numbers, mappings and projections (including stereographic projection and bilinear transformation).

Elementary and analytic functions: functions of complex variables, examples of elementary functions like exponential, trigonometric and hyperbolic functions, elementary calculus on the complex plane (limits, continuity, differentiability), Cauchy Riemann equations, analytic functions, harmonic functions with examples, branch points and branch cuts, multi-valued functions (eg. logarithmic function and its branches, Riemann surfaces).

Complex integration: Cauchy's integral theorem, Cauchy integral formula for higher derivatives, Morera's theorem, Liouville's theorem, maximum-modulus principle, Schwarz lemma.

Series expansion of complex functions: power series, Taylor and Laurent series of complex functions, convergence, definition of holomorphic and meromorphic functions, zeros and poles, classification of singular points, removable singularities, Weierstrass theorems (M test and factor theorem).

Residue calculus: general form of Cauchy's theorem, Cauchy residue theorem, evaluation of definite integrals using residue theorem (principal value integrals and integrals with branch points), argument principle and Roche's theorem (eg. with application to prove the fundamental theorem of algebra), residue at infinity.

Conformal maps: elementary conformal maps (Schwarz-Christoffel transformation), analytic continuation, method of analytic continuation by power series (eg. application in defining the Riemann-Zeta function).

==

Course Learning Outcomes (CLO): Upon the completion of this course, the students will obtain conceptual skills to practise the following mathematical techniques.

1. Representation of complex numbers in Cartesian, polar and matrix form, geometrical interpretation of complex numbers.
2. Analyticity of complex functions including the utility of Cauchy-Riemann equations, evaluation of contour integrals using Cauchy integral formula.
3. Representation of complex functions as power series (eg., Taylor and Laurent) and their convergence, classification of singularities.
4. Application of residue calculus using Cauchy's residue theorem, method of analytic continuation.
5. Elementary understanding of conformal maps.

==

Recommended Books:

1. Ablowitz, M. and Fokas, A. S., *Complex Variables: introduction and applications*, Cambridge University Press, 2003 (2nd edition).
2. Churchill, R.V. and Brown J.W., *Complex Variable and Applications*, McGraw Hill, 2009 (8th edition).
3. Ahlfors, L.V., *Complex Analysis*, Tata McGraw Hill, 1979 (3rd edition).
4. Kasana, H.S., *Complex Variables: Theory and Applications*, Prentice Hall India, 2005 (2nd edition).
5. Ponnuswamy, S., *Foundation of Complex Analysis*, Narosa Publishing House, 2011 (2nd edition).

==

Course evaluation scheme:

1. Mid-semester test (30%)
2. End-semester test (45%)
3. Sessionals (25%)

==