Introduction to hypothesis tests. (Principles of inference).

Goal :-

i) Use data from a sample to make inference about the population from which the sample was drawn

ii) Sampling D^n's like χ^2, F, t, $N(\mu, \sigma^2)$ play a pivotal role.

Statistical Inference.

- a <u>Statement (hypothesis)</u> about the parameter (that we are interested in estimating about the population)
- a <u>measure</u> of the reliability of that Statement in terms of a probability.

What is a hypothesis?

→ A hypothesis usually results from speculation concerning observed behavior, natural phenomena or established theory.

If a hypothesis is stated in terms of population parameters such as the mean & variance, then it's called a <u>statistical hypothesis</u>.

Data from a sample is used to test the validity of the hypothesis.

Components of hypothesis testing.

→ Null (H_0) & Alternate (H_1 or H_a) hypothesis

→ Decision making / Rejection (Critical) region.

→ Errors in decision

Types of error → α (Type I error probability)
→ β (Type II error prob.).

* We would like to minimize these errors in the manner we make a decision about our hypothesis!

the General Mantra

5-step process for hypothesis testing.

Step 1 :- Specify H_0, H_1 and an acceptable level of α

combined step

Step 2 :- Define a sample based test statistic (eg $\bar{X}$, S^2, etc) & the rejection region for H_0

Step 3 :- Collect the sample data & Calculate the test statistic

Step 4 :- Make a decision to either reject or fail to reject H_0.

Step 5 :- Interpret the result in the language of the problem (i.e provide confidence interval, type & probability of error in the decision etc).

A simple hypothesis test.

This is an application of $Z \sim N(0,1)$ distribution. This example is relevant to quality control in a packaging industry.

Q) A company that packages salted peanuts in 8 kg jars is interested in maintaining control on the amount of peanuts put in jars by one of its machines. Control is defined as averaging 8 kg per jar & not consistently over/underfilling the jars. To monitor this control, a sample of 16 jars is taken from the packaging line at random time intervals & their contents weighed. The mean weight of peanuts in these 16 jars (test statistic) will be used to test the null hypothesis that the machine is indeed working properly. If it is found not to be doing so, an expensive adjustment will be req'd.

soln :-

Step 1 :-

Hypothesis :- $H_0 : \mu = 8$

$H_1 : \mu \neq 8$

set α.

Step 2 & 3 :-

Test statistic $\longrightarrow$ $\bar{X} = \dfrac{\sum_{i=1}^{16} x_i}{16}$

Step 4 :- Rejection region (Critical) eg. $\bar{X} < 7.9$ or

$\bar{X} > 8.1$

Step 5 :- Type I error / Type II error. What are these errors?

Errors!

Decision	H_0 is TRUE	H_0 is NOT TRUE
H_0 is NOT rejected	Decision is Correct	type II error (w/ probability β)
H_0 is rejected	type I error (w/ prob. α)	Decision is Correct

in the population

How to compute probability of error?

CLT

$$\alpha = \text{Prob}\left(\bar{X} < 7.9 \text{ or } \bar{X} > 8.1 \text{ when } \mu = 8\right)$$

$n=16$

Assume (for now) that we somehow know σ of the population (of all jars) to be 0.2 (this will often not be the case, & t-D^n will be req'd for analysis).

$$P(\bar{X} < 7.9) = P\left(\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} < \frac{7.9 - 8}{0.2/\sqrt{16}}\right) = P(Z < -2.0)$$

$$= 0.0228.$$

Likewise

$$P(\bar{X} > 8.1) = P\left(\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} > \frac{8.1 - 8}{0.2/\sqrt{16}}\right)$$

$$= P(Z > 2.0)$$

$$= 0.0228.$$

$$\therefore \alpha = \text{prob(type I error)} = P(\bar{X} < 7.9) + P(\bar{X} > 8.1)$$

$$= 0.0228 + 0.0228$$

$$= 0.0456.$$

— x —