Linear algebra problem sheet (Module-3)

February 6, 2022

1 Linear Differential Equation with Constant Coefficients

1. Find the general solution of $y''(t) + y'(t) - 2y(t) = 0$
2. Show that if y_1 and y_2 are solutions of second order linear differential equation with constant coefficient then their linear combination is also a solution.
3. Find the solution of given differential equation $y'' - 5y' + 6y = 0$, $y(1) = e^2$ and $y'(1) = 3e^2$
4. Solve the ODE $y^{(4)}(t)$=0
5. For given differential equation write down the basis of solution space $y''(t) - 10y'(t) + 25y(t) = 0$
6. Find the particular solution of given ODE using the Method of Undermined Coefficient. $y'' + y = \sin x$
7. Find the general solution of ODE $y'' - y' - 2y = 4x^2$
8. Solve $y''' - 3y'' + 3y' - y = e^t$
9. solve $y'' - 3y' + 2y = 14\sin 2x - 18\cos 2x$
10. If k and b are positive constants find the general solution of $y'' + k^2 y = \sin bx$

2 System Of First Order Linear Differential Equation

11. Solve the given system of linear equation

$$\frac{dx}{dt} = x + y \tag{1}$$

$$\frac{dy}{dt} = 4x - 2y \tag{2}$$

12. Solve the given system of linear equation

$$\frac{dx}{dt} = 4x - 5y \tag{3}$$

$$\frac{dy}{dt} = 5x - 4y \tag{4}$$

13. Convert the second order differential equation in question(3) into two first order system of differential then solve.

14. Describe the phase portrait of given system of equations

$$\frac{dx}{dt} = 1 \tag{5}$$

$$\frac{dy}{dt} = 2 \tag{6}$$

15. Draw the phase portrait of given system also draw the nullclines.

$$\frac{dx}{dt} = -x \tag{7}$$

$$\frac{dy}{dt} = -y \tag{8}$$

16. Discuss the stability of critical point of question (15)

17. Find the critical points of given system of first order ode and discuss the stability

$$\frac{dx}{dt} = 4x - 2y \tag{9}$$

$$\frac{dy}{dt} = 5x + 2y \tag{10}$$

18. Solve the given system of ODE

$$\frac{dx}{dt} = 2x - 2y + 10 \tag{11}$$

$$\frac{dy}{dt} = 11x - 8y + 49 \tag{12}$$

19. Using Wronskian show that given functions are linearly independent, $y_1(x) = 1$,$y_2(x) = x$,$y_3(x) = x^2$

20. Let $y_1(x) = x^3$ and $y_2(x) = x^2|x|$ where x belongs to R show that their Wronskian is zero but they are linearly independent