Linear algebra problem sheet (Module-2)

January 20, 2022

Gram Schmidt Orthogonalization

1. Consider the basis $B = \{v_1, v_2, v_3\}$ of R^3 where $v_1 = (3, 0, 4)$, $v_2 = (-1, 0, 7)$, $v_3 = (2, 9, 11)$ then using Gram Schmidt process find corresponding orthonormal basis.

2. Perform the Gram–Schmidt process on the following basis vector of R^2

$$v_1 = \begin{bmatrix} -3 \\ 4 \end{bmatrix} \tag{1}$$

$$v_2 = \begin{bmatrix} 1 \\ 7 \end{bmatrix} \tag{2}$$

   illustrate your work geometrically.

3. Find the QR factorization of the matix
$$\begin{bmatrix} 2 & -2 & 18 \\ 2 & 1 & 0 \\ 1 & 2 & 0 \end{bmatrix}$$

Eigenvalues,Eigenvectors and Diagonalization

4. Find all eigenvalues and eigenvectors of n*n identity matrix.

5. Find the eigenvalues of rotation matrix of $\pi/2$ in R^2.

6. Find all 2×2 matrices for which $v_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ is eigenvector with associated eigenvalue 5 .

7. If A is invertible matrix such that λ is eigen value and v is associated eigenvector,then find eigenvalue and associated eigenvector of A^{-1}.

8. Find all the eigenvalues of given matrix $\begin{bmatrix} 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \end{bmatrix}$

9. Let A be 2*2 matrix such that trace(A)= 9 and det(A)=17/4,then find all the eigenvalues of A.

10. How many 2*2 matrices A, are possible for which trace(A) =0 and det(A) =0.

11. Find eigenvalues of the matrix . $\begin{bmatrix} 1 & 2 & 5 \\ 0 & -3 & 4 \\ 0 & 0 & 6 \end{bmatrix}$

12. If 1,3 and 5 are eigenvalues of a 3*3 matrix A, then find trace of A^{-1}

13. For the matrix in question (11)find the eigenbasis.

14. Find the algebraic and geometric multiplicities of eigenvalues of 3*3 identity matrix.

15. Check whether the given matrix is diagonalizable or not ,$A = \begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix}$

16. Is n*n identity matrix diagonalizable? If yes then what type of matrix can be used to diagonalize it?

17. Diagonalize the given matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$

18. Let A be 3*3 matrix such that 2,2 and 3 are the eigenvalues of A ,if rank(A-2I)=2, where I is 3*3 identity matrix .Is this matrix diagonalizable?

19. Diagonalize the given matrix(Checking the diagonalizability conditions) $\begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$

20. Find eigenbasis of matrix A and diagonalize it $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$