Name five application problems from engineering and science that we have discussed in this entire course?

(i) [redacted]

(ii) [redacted]

(iii) [redacted]

(iv) [redacted]

(v) [redacted]

|Notes| (1) Date 5/12/23

Sketching Solution Trajectories of ODEs

Slope fields: It consists of "short lines" representing slopes (steepness) sketched at lots of different pts.

→ graphical approach to trace solution trajectories of ODEs

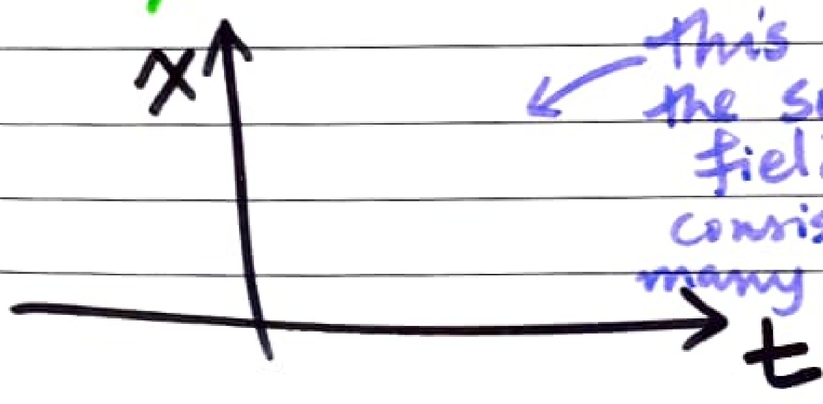
eg.

(1) $\frac{dx}{dt} = t - 2x$

(a) Identify indep. variable: t

(b) Id. dependent variable: x

(c)



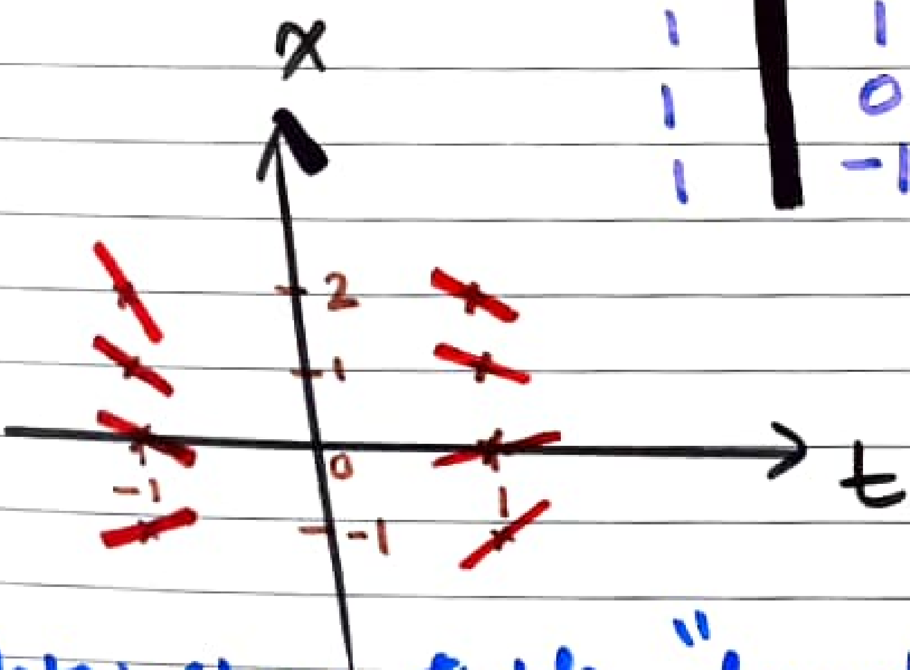
← This is the sol^n field consisting many pts...

(d) Identify ordered pairs in the soln field and find slopes $\left(\frac{dx}{dt}\right)$ at each of those ordered pairs. Tabulate!

Recall:

$$\frac{dx}{dt} = t - 2x$$

t	x	$\frac{dx}{dt}$
-1	2	-5
-1	1	-3
-1	0	-1
-1	-1	1
1	2	-3
1	1	-1
1	0	1
1	-1	3

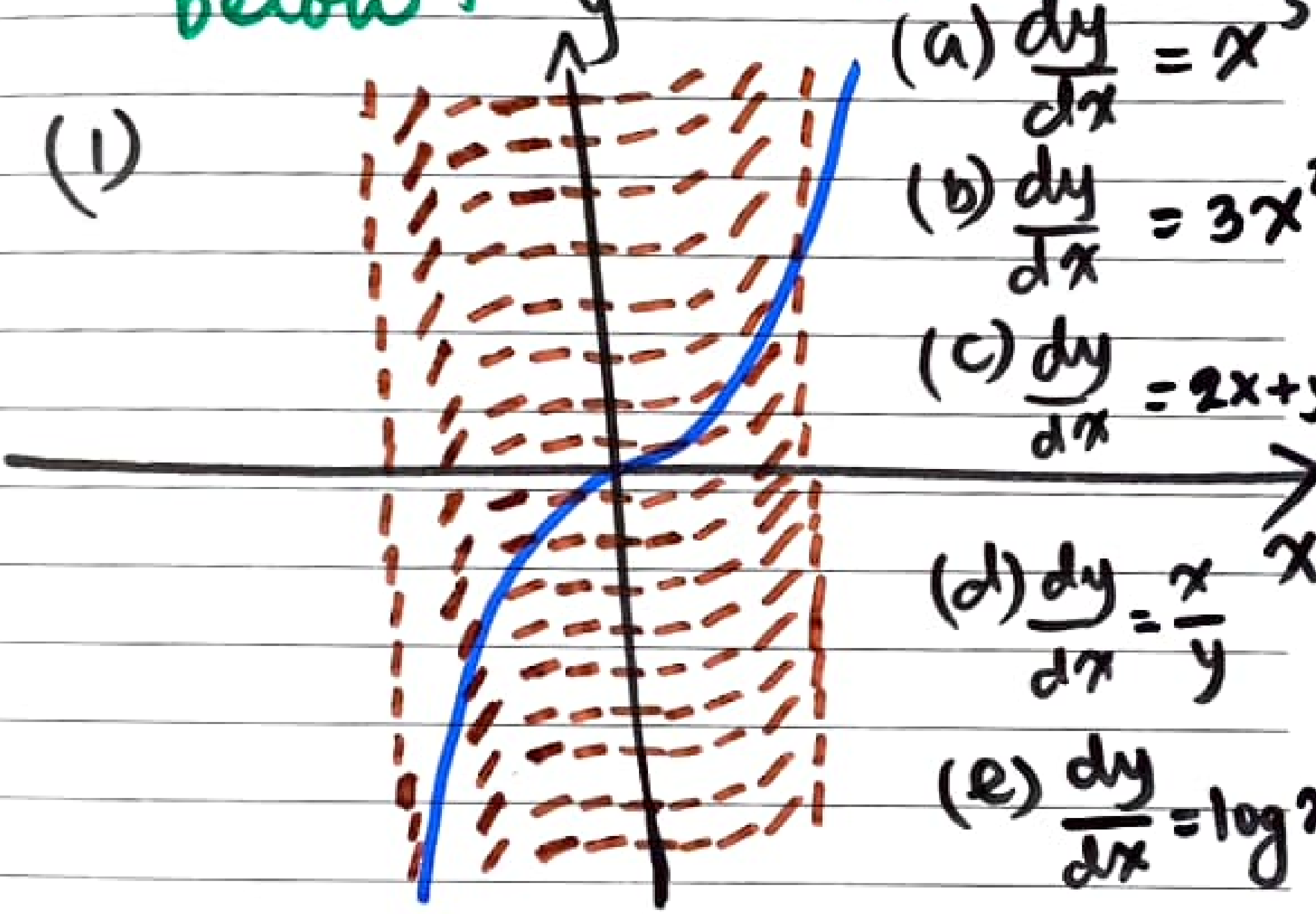


Note: slope fields "do <u>not</u>" have arrows.

eg

(2) Determine the differen... eq^n being graphed by each of the slope fields below

(1)



(a) $\frac{dy}{dx} = x^3$

(b) $\frac{dy}{dx} = 3x^2$

(c) $\frac{dy}{dx} = 2x+y$

(d) $\frac{dy}{dx} = \frac{x}{y}$

(e) $\frac{dy}{dx} = \log x$

Try to trace out a soln!

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Ans:-

(b) $\frac{dy}{dx} = 3x^2$

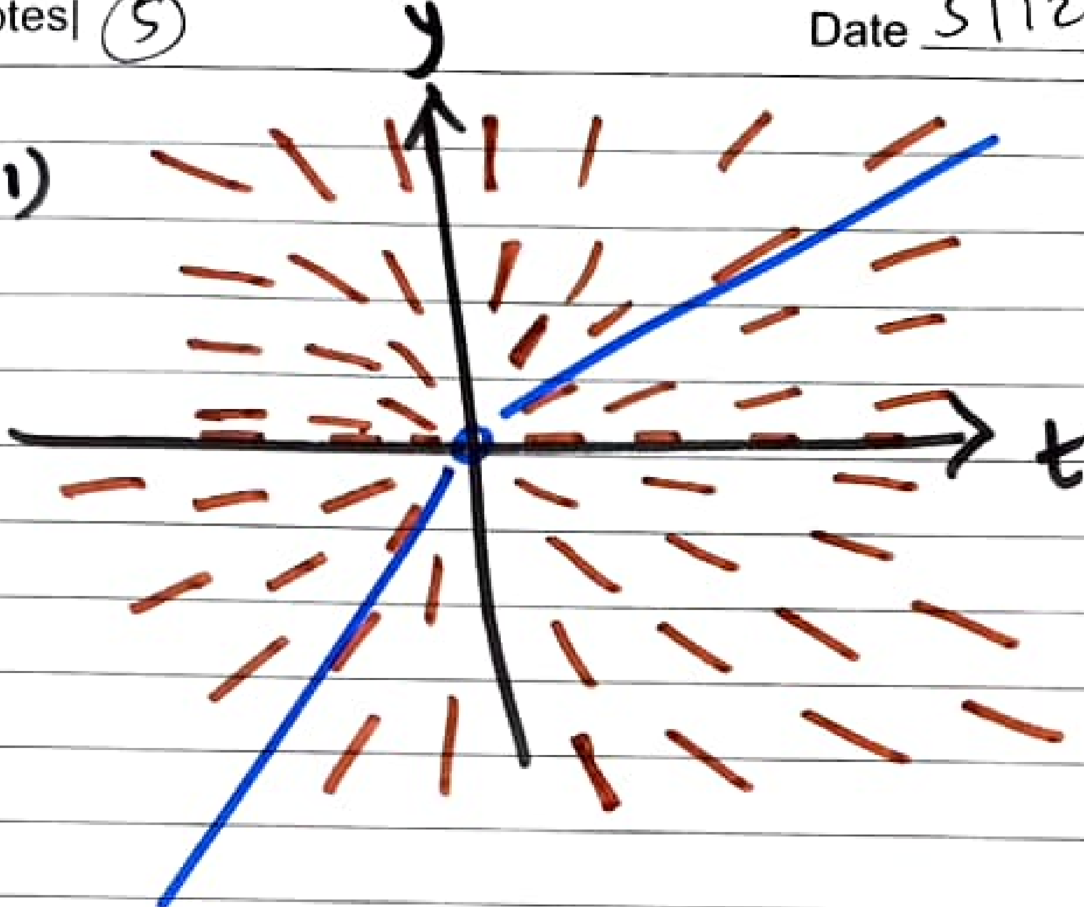
- All slopes are +
- $\frac{dy}{dx} = 3x^2$

$\int dy = \int 3x^2 dx$

$y = x^3 + c$

↑ family of f^ns that traces the sol^ns.

(11)



(a) $\frac{dy}{dt} = t - 2$

(b) $\frac{dy}{dt} = t^3$

(c) $\frac{dy}{dt} = t - y$

(d) $\frac{dy}{dt} = \frac{y}{t}$

(e) $\frac{dy}{dt} = e^y$

Ans:-

(d) $\frac{dy}{dt} = \frac{y}{t}$

- ordered pairs on y-axis have ∞ slope
- ordered pairs on t(x)-axis have 0 slope
- $\int \frac{dy}{dt} = \int \frac{y}{t}$

$e^{\log|y|} = e^{\log|t| + c}$

$= e^{\log t} e^{c}$

$y = Kt$ (linear soln. trajectories)

Sol^n trajectories (& dir fields) on the phase plane – sys. of ODEs (2x2 case)

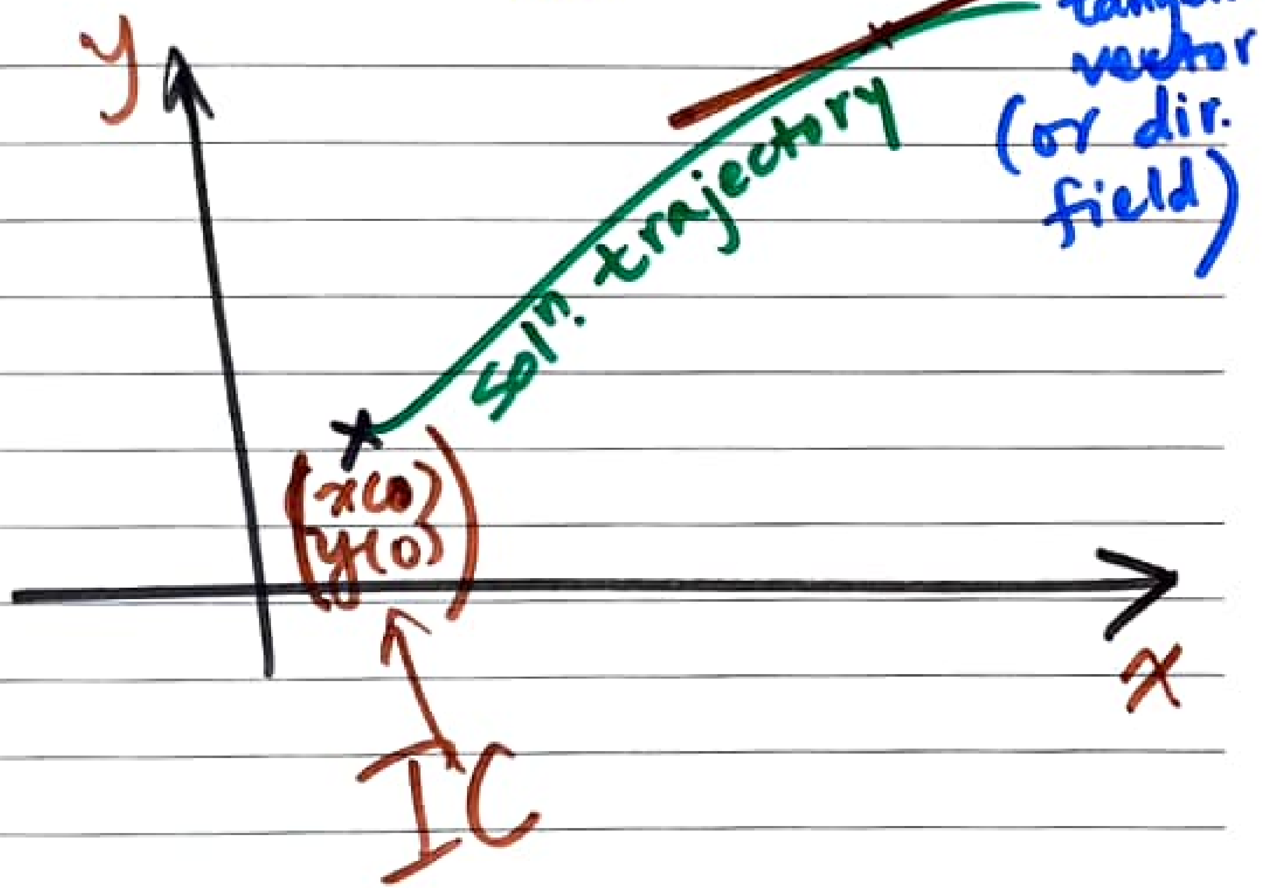
PHASE PORTRAITS

$$\frac{dx}{dt} = f(x, y; t)$$

$$\frac{dy}{dt} = g(x, y; t)$$

Sol^n: $\begin{pmatrix} x(t) \\ y(t) \end{pmatrix}$ is a parametric curve on the x-y (phase plane)

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$



Phase portrait: Collection of soln. trajectories corresponding to various ICs.

eg (1)

$$\vec{\psi}' = A\psi$$

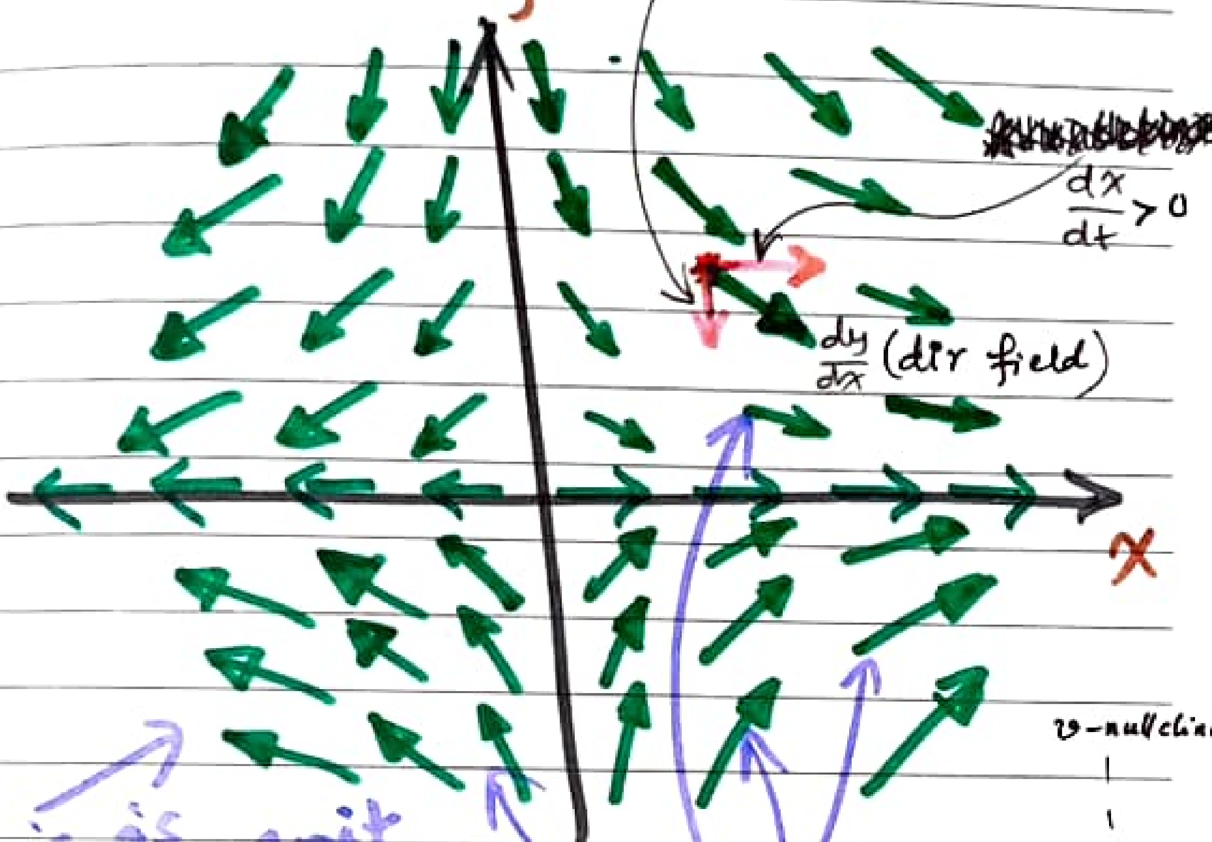
$$\begin{pmatrix} x \\ y \end{pmatrix}' = \begin{pmatrix} 2 & 0 \\ 0 & -3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

i.e. $$\begin{pmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \end{pmatrix} = \begin{pmatrix} 2x \\ -3y \end{pmatrix}$$

eq^{m} pt. (0, 0)

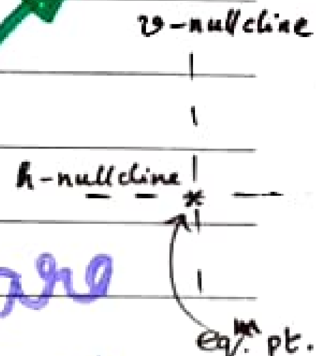
eVs	EVs
2 (unstable)	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$
−3 (stable)	$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

↑ real eVs ↑ real EVs



This is a ph. portrait

These are all $\frac{dy}{dx}$ at some pt (x, y)



The sys. of DE can be written as:

$$\frac{dy}{dx} = -\frac{3}{2}\frac{y}{x}$$

(x, y)	$\frac{dy}{dx}$
(1, 10)	−15
(1, 6)	−9
(1, 2)	−3
(2, 1)	−0.75
(10, 1)	−0.15

$\frac{dx}{dt} = 0$	$\frac{dy}{dt} = 0$
x=0	y=0

The dir field dy/dx is a "resultant" vector drawn by 1st sketching out the v-nullcline $\left(\frac{dx}{dt} = 0\right)$ and h-nullcline $\left(\frac{dy}{dt} = 0\right)$

Steps to sketch ph. portraits:

(i) Identify v- nullclines and h- nullclines

(ii) Intersection pt(s) of v- & h- nullclines are eq^m pts.

(iii) stable/unstable eq^m pts.
$\lambda > 0$ or $\lambda < 0$

(iv) find EVs
Sketch separatrix

(v) draw dir field $\left(\frac{dy}{dx}\right)$
by tabulating $\frac{dy}{dt}$, $\frac{dx}{dt}$

(vi) sketch some sol^n trajectories w.r.t. ICs.

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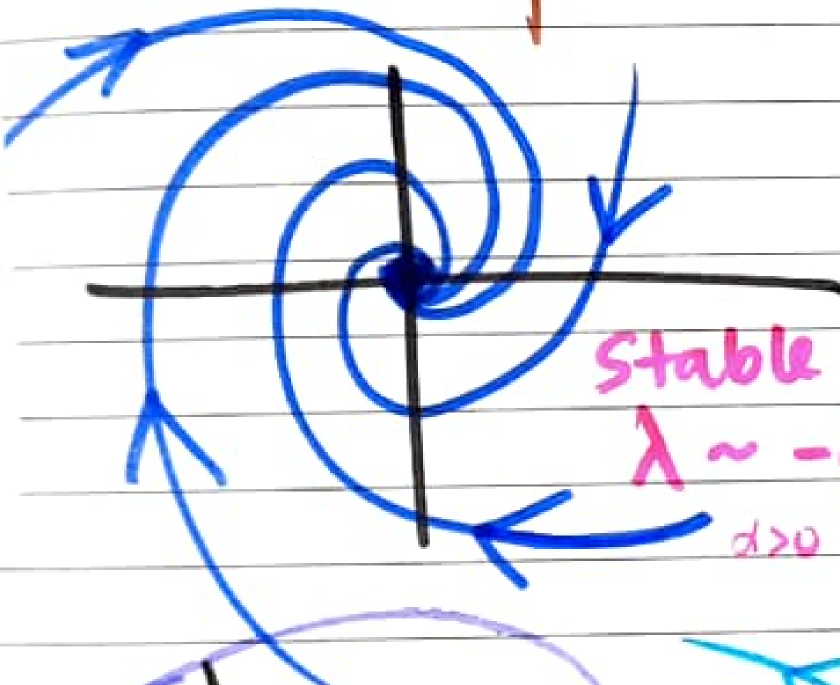
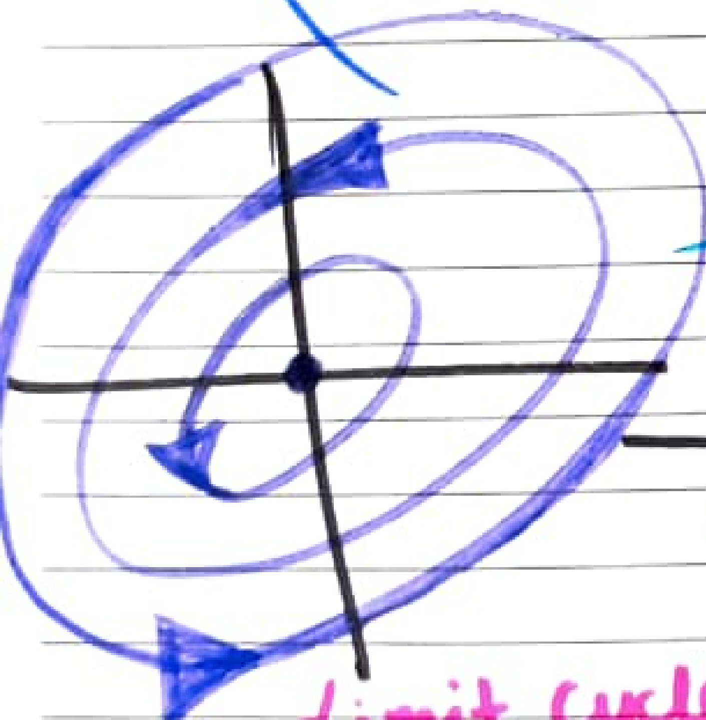
How do we draw ph. portraits when EVs (and EVs) are complex no.s?

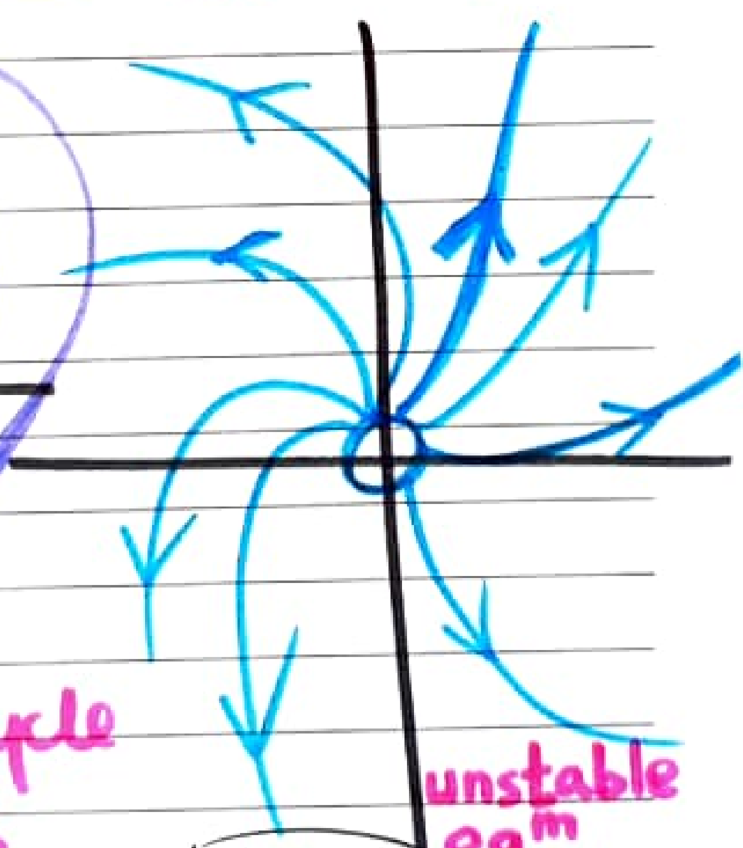
← Can NOT represent these on the phase plane



stable eq^m
$\lambda \sim -\alpha \pm i\beta$
$\alpha > 0$



Limit cycle (stable)
$\lambda \sim \pm i\beta$



unstable eq^m
$\alpha > 0$
$\lambda \sim \alpha \pm i\beta$