

**Instructions:** Each group is assigned one problem. Within each group, you may work in teams but each of you must write and submit your own solutions. Violators may be penalized up to the maximum credit for the assignment. You must turn in your solutions as a scanned copy via email to amriksen@thapar.edu by 9 pm of April 18, 2020. Maximum credit for this assignment is five points.

1. **Group 1:**

If Cauchy Riemann conditions are satisfied by  $f(z)$ , then what can you say about  $\frac{\partial f}{\partial \bar{z}}$ ? Here  $\frac{\partial}{\partial \bar{z}} := \frac{1}{2}(\partial_x + i\partial_y)$ . Why?

2. **Group 2:**

Besides the Cauchy Riemann conditions, can you suggest another way to check if a function is analytic or not? Give a one line answer in no more than 10 words.

3. **Group 3:**

In the lecture notes on the min-max principle, as an application to PDE (or harmonic function) theory, it is stated that if  $|g(z)|$  attains its maximum on  $\partial D$ , then  $u(x, y)$  attains its maximum on  $\partial D$  and likewise for  $|h(z)|$  and  $v(x, y)$ . Please provide technical (mathematical) reason(s) to support the above statements.

4. **Group 4:**

For the problem in example 2.5.2 of pg. 88 in your textbook that was discussed in detail in the online video lectures, what does  $I$  evaluate to when  $m = -1$ ? Why?

5. **Group 5:**

Let  $C(t)$  be a cycle s.t.  $C(t) := e^{ikt}$ , for  $0 \leq t \leq 2\pi$ , where  $k$  is a positive integer. Then evaluate  $\frac{1}{2\pi i} \oint_{C(t)} \frac{dz}{z}$ ? What is the geometrical significance of this result?

*Hint: What is the meaning of  $k$  in the definition of  $C(t)$ ?*

6. **Group 6:**

Compute  $\frac{1}{2\pi i} \oint_C \frac{\zeta}{\zeta-z} d\zeta$ , where  $C := |\zeta| = 1$  is the unit circle.

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