Tutorial 5
Residue calculus, principal value integrals & conformal mappings

1. Evaluate $I = \int_{-\infty}^{\infty} \frac{\cos kx}{(x+b)^2+a^2}$, $k > 0,\ a > 0,\ b \in \mathbb{R}$.

2. Evaluate $I = \int_0^{2\pi} \frac{d\theta}{A+B\sin\theta}$, where $A^2 > B^2,\ A > 0$. Hence find $I = \int_0^{2\pi} \frac{d\theta}{A+B\cos\theta}$ without doing any calculations.

3. Evaluate $I = \int_{-\infty}^{\infty} \frac{e^{px}}{1+e^x} dx$ for $0 < \mathrm{Re}\{p\} < 1$.

4. Suppose that on a contour C_ϵ that subtends an angle ϕ at the point z_0, we have $(z - z_0)f(z) \to 0$ uniformly as $\epsilon \to 0$, then show that $\lim_{\epsilon\to 0} \int_{C_\epsilon} f(z)dz = 0$. Further, if $f(z)$ has a simple pole at z_0 with $\mathrm{Res}(f(z); z_0) = c_{-1}$, then for the contour C_ϵ, show that $\lim_{\epsilon\to 0} \int_{C_\epsilon} f(z)dz = i\phi c_{-1}$.

5. Show that $\int_{-\infty}^{\infty} \frac{\sin ax}{x} dx = sgn(a)\pi$ by evaluating the principal value integral $⨍_{-\infty}^{\infty} \frac{e^{iax}}{x} dx$, $a \in \mathbb{R}$.

6. Show that $w = f(z) = z^2$ maps horizontal and vertical grid lines to mutually orthogonal parabolas. (The conformal map preserves the right angles between the grid lines.)

7. Find a map that inverts a unit circle, i.e. maps the inside of the circle to the outside. Will the arrows on the curves be reversed?

□