Finding inverse of a matrix using the Gauss-Jordan elimination method

Example 1

Suppose that we want the inverse of

$$S = \begin{pmatrix} 2 & 4 & -2 \\ 4 & 9 & -3 \\ -2 & -3 & 7 \end{pmatrix}.$$

We can use Gaussian Elimination to solve the systems $SX_{:,1} = e_1, SX_{:,2} = e_2, SX_{:,3} = e_3$ for the three columns of $X = S^{-1}$

$$\left(\begin{array}{rrr|rrr} 2 & 4 & -2 & 1 & 0 & 0 \\ 4 & 9 & -3 & 0 & 1 & 0 \\ -2 & -3 & 7 & 0 & 0 & 1 \end{array}\right) \to \left(\begin{array}{rrr|rrr} 2 & 4 & -2 & 1 & 0 & 0 \\ 0 & 1 & 1 & -2 & 1 & 0 \\ 0 & 1 & 5 & 1 & 0 & 1 \end{array}\right)$$

$$\to \left(\begin{array}{rrr|rrr} 2 & 4 & -2 & 1 & 0 & 0 \\ 0 & 1 & 1 & -2 & 1 & 0 \\ 0 & 0 & 4 & 3 & -1 & 1 \end{array}\right)$$

$$\to \left(\begin{array}{rrr|rrr} 2 & 4 & 0 & 5/2 & -1/2 & 1/2 \\ 0 & 1 & 0 & -11/4 & 5/4 & -1/4 \\ 0 & 0 & 1 & 3/4 & -1/4 & 1/4 \end{array}\right) \to \left(\begin{array}{rrr|rrr} 1 & 0 & 0 & 27/4 & -11/4 & 3/4 \\ 0 & 1 & 0 & -11/4 & 5/4 & -1/4 \\ 0 & 0 & 1 & 3/4 & -1/4 & 1/4 \end{array}\right).$$

$$S^{-1} = \frac{1}{4}\begin{pmatrix} 27 & -11 & 3 \\ -11 & 5 & -1 \\ 3 & -1 & 1 \end{pmatrix}.$$

Example 2

Suppose that we want the inverse of

$$S = \begin{pmatrix} 2 & 3 & -2 \\ 1 & -2 & 3 \\ 4 & -1 & 4 \end{pmatrix}.$$

We can use Gaussian Elimination to solve the systems $SX_{:,1} = e_1, SX_{:,2} = e_2, SX_{:,3} = e_3$ for the three columns of $X = S^{-1}$

$$\left(\begin{array}{rrr|rrr} 2 & 3 & -2 & 1 & 0 & 0 \\ 1 & -2 & 3 & 0 & 1 & 0 \\ 4 & -1 & 4 & 0 & 0 & 1 \end{array}\right) \to \left(\begin{array}{rrr|rrr} 2 & 3 & -2 & 1 & 0 & 0 \\ 0 & -7/2 & 4 & -1/2 & 1 & 0 \\ 0 & -7 & 8 & -2 & 0 & 1 \end{array}\right)$$

$$\to \left(\begin{array}{rrr|rrr} 2 & 3 & -2 & 1 & 0 & 0 \\ 0 & -7/2 & 4 & -1/2 & 1 & 0 \\ 0 & 0 & 0 & -1 & -2 & 1 \end{array}\right).$$

None of the linear systems $SX_{:,1} = e_1, SX_{:,2} = e_2, SX_{:,3} = e_3$ has a solution. Therefore, S is not invertible.