Rubric for Project-3, Total- 10 marks.

1 Notations

We recall the notations which were introduced in class to understand the Viterbi Algorithm.

i) State space is $S = \{s_1,\ s_2, ..., s_K\}$. The Markov process is denoted as X which is a stochastic process, i.e, at each time step t, it is a random variable which takes the values say x_t. So, we write $X = \{x_1, ..., x_T\}$.

ii) Space of possible observation, $O = \{o_1, ..., o_N\}$. We denote the observations given to us as the Markov process Y. Note that Y is also a stochastic process and we write $Y = \{y_1, ..., y_T\}$ (where y_t is the observation made at the time t and $y_t \in O$). Initially, we are given values of y_t for each times step and using this information we will predict the most likely path $x_1 \to x_2 \to x_3 \to \cdots \to x_T$.

iii) We write the initial probabilities as $\Pi = \{\pi_1, ..., \pi_K\}$ where π_n denotes the probability of being in state s_n at $t = 1$.

iv) The $K \times K$ probability transition matrix $\mathbb{P}$ which has the entries

$$p_{ij}(t) = P(x_t = s_j \mid x_{t-1} = s_i), \text{ and } \mathbb{P} = \begin{array}{c} \\ s_1 \\ s_2 \\ \\ s_K \end{array} \begin{array}{c} \begin{array}{cccc} s_1 & s_2 & \cdots & s_K \end{array} \\ \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1K} \\ p_{21} & p_{21} & \cdots & p_{2K} \\ \vdots & & \ddots & \vdots \\ p_{K1} & p_{K2} & \cdots & p_{KK} \end{bmatrix} \end{array}.$$

v) The $K \times N$ emission matrix $\mathbb{E}$ which has the entries

$$e_{ij}(t) = P(y_t = o_j \mid x_{t-1} = s_i), \text{ and } \mathbb{E} = \begin{array}{c} \\ s_1 \\ s_2 \\ \\ s_K \end{array} \begin{array}{c} \begin{array}{cccc} o_1 & o_2 & \cdots & o_N \end{array} \\ \begin{bmatrix} e_{11} & e_{12} & \cdots & e_{1N} \\ e_{21} & e_{21} & \cdots & e_{2N} \\ \vdots & & \ddots & \vdots \\ e_{K1} & e_{K2} & \cdots & e_{KN} \end{bmatrix} \end{array}.$$

vi) Recall that to simplify our notation, we write $s_i = i$ and $o_j = j$ where it must be understood that $x_t = i$ refers to the Markov process X taking the state s_i at time t and $y_t = j$ refers to the Markov process Y taking the state o_j at time t.

2 Questions

For questions refer to 'this'. See below for marks breakup. It is same for both the questions. So total is $5 + 5 = 10$.

(i)

Give the correct $K \times T$ Viterbi Probabilities matrix. Each entry is denoted by $V_{k,t}$. (2 marks)

The matrix entry $V_{k,t}$ stores the probability of the most likely path ending in state x_k at time t.

(ii)

Give the correct $K \times T$ Viterbi Path matrix $Z_{K,t}$. (2 marks)

The matrix entry $Z_{k,t}$ stores the value of the state at the previous time step $(t-1)$ which was along the most likely path ending in state x_k at time t.

(iii) Clearly state the value taken by the random variables $x_1, ..., x_K$ when along the most probable path

$$x_1 \to x_2 \to \cdots \to x_K \quad \text{(1 mark)}$$

Important Note:

(a) In (i) or (ii) if one entry of either Viterbi probability/Viterbi path matrix is incorrect, you lose 1 mark. If more than 1 entry is incorrect, you won't get any credits (i.e, 2 marks) for that part.

(b) If any of the entries in the path x_t (for (iii)) is incorrect, you lose the complete credit (1 mark).

(c) You will only be allotted the complete score for any part above subject to satisfactorily answering the questions asked during your interview by TA's.

(d) It is not possible to entertain individual requests from students to be assessed by a specific TA. The grader for each student would be randomly allotted and any change would not be entertained.