Sampling Distributions.

① χ^2 distribution :- describes the distribution of sample variance.

(1.1) Defⁿ :- $Z_i \sim N(0,1)$; $i = 1, 2, \dots, n$

$$\chi^2(n) \sim \sum_{i=1}^{n} Z_i^2 \quad ; \quad n = \text{no. of degrees of freedom}$$

Sometimes written as $\chi^2(\nu)$ where $\nu = n$

(1.2) Properties of χ^2 Dⁿ.

(i) χ^2 values > 0
(ii) Shape of χ^2 Dⁿ is different for different values of ν
(iii) $Y \sim \chi^2(\nu)$
$E(Y) = \nu$; $Var(Y) = 2\nu$
(iv) for $\nu \geq 30$; $\chi^2(\nu) \xrightarrow{\text{approx}^n} N(\nu, 2\nu)$
$$\Rightarrow Z = \frac{\chi^2 - \nu}{\sqrt{2\nu}} \sim N(0,1)$$

(1.3) Application of χ^2 D^n

D^n of sample variance :-

$$Y_i \sim N(\mu, \sigma^2) \; ; \quad i = 1, 2, \ldots, n$$

then $\dfrac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1)$ where $S^2 = \dfrac{\sum_{i=1}^{n}(Y_i - \bar{Y})^2}{n-1}$ is the "unbiased" sample variance.

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2 important theorems :-

* **Sampling D^n of the mean** :- the sampling D^n of $\bar{Y}$ from a random sample of size n drawn from a population w/ mean μ & variance σ^2 will have mean μ and variance $\dfrac{\sigma^2}{n}$.

** Central Limit th^m (CLT) :-

If random samples Y_i of size n are taken from any D^n w/ mean μ and variance σ^2; then

$$\bar{Y} \sim N\left(\mu, \frac{\sigma^2}{n}\right) \text{ as } n \to \infty .$$

② t - D^n :-

(2.1) Def^n $$Z = \frac{\bar{Y} - \mu}{\sigma/\sqrt{n}} \sim N(0,1)$$

But often the population ~~size~~ std. dev. (σ) is unknown.
If σ is replaced by the sample std dev s;
then Z is no more $N(0,1)$.

This led to the formulation of the t - D^n by Gosset w/ ν degrees of freedom

$$t(\nu) = \frac{Z}{\sqrt{\frac{\chi^2(\nu)}{\nu}}}$$; $Z \sim N(0,1)$ χ^2 is an indep- RV.

(2.2) Application.

deg. of freedom

$\because Z = \dfrac{\bar{Y} - \mu}{\sigma/\sqrt{n}} \sim N(0,1)$

and $\chi^2(n-1) = \dfrac{(n-1)s^2}{\sigma^2}$ has χ^2 D^n w/ $(n-1)$ d.o.f.

$$T = \frac{Z}{\sqrt{\frac{\chi^2(n-1)}{n-1}}} = \frac{\bar{Y} - \mu}{s/\sqrt{n}} \sim t(n-1)$$

(2.3). Properties :-

$t(\infty) \sim N(0,1)$.

§(3) F - D^n (named after Sir Ronald Fisher).

§(3.1) Def^n

$$F(\nu_1, \nu_2) = \frac{\frac{\chi_1^2(\nu_1)}{\nu_1}}{\frac{\chi_2^2(\nu_2)}{\nu_2}}$$

; χ_1^2 and χ_2^2 are independent of each other.

$\nu_i = (n_i - 1)$

Also if we have a sample of size n_i; $i = 1, 2, \ldots$ from a population w/ variance σ_i^2; $i = 1, 2, \ldots$ each sample being independent of the other

$$F = \frac{S_1^2/\sigma_1^2}{S_2^2/\sigma_2^2} \; ;$$

S_i^2 is the variance estimate of σ_i^2 ; $i = 1, 2$

§(3.2) <u>properties of F - D^n</u> :-

(i) F D^n is defined for non-negative values.

(ii) Not - Symmetric in shape.

§(4) <u>Relationship among the D^ns</u>.

(i) $t(\infty) = Z \sim N(0,1)$

(ii) $Z^2 = \chi^2(1)$

(iii) $F(1, \nu_2) = t^2(\nu_2)$

(iv) $F(\nu_1, \infty) = \dfrac{\chi^2(\nu_1)}{\nu_1}$

Read "$\sim$" as " has the D^n "