Q1) **Using the bisection method find the approximate value of square root of 3 in the interval (1, 2) by performing two iterations.**

Solution:

Let $x = \sqrt{3}$,

Squaring both the sides we have,

$x^2 = 3$

$\therefore x^2 - 3 = 0$

The positive root of this equation is $\sqrt{3}$.

Let $f(x) = x^2 - 3$

$\therefore f(1) = (1)2 - 3 = 1 - 3 = -2 < 0$ (negative)

$\therefore f(2) = (2)2 - 3 = 4 - 3 = 1 > 0$ (positive)

$\therefore$ By intermediate value theorem, the root of equation $f(x) = 0$ lies in interval

$(1, 2) = (a, b)$

- **Initial approximation : (First approximation)**

Let x_o be the initial approximation (first approximation) to the root.

By the bisection formula

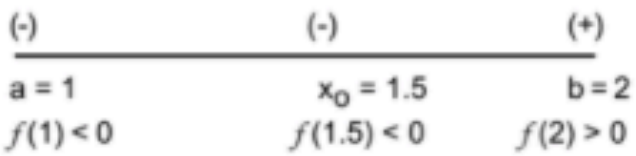
$x_o = (a + b)/2 = (1 + 2)/2 = 3/2 = 1.5$

- **First Iteration (Second approximation) :**

Now, $f(1.5) = (1.5)^2 - 3$

$= 2.25 - 3$

$= -0.75 < 0$ (negative)



Now, $f(1.5) < 0$ and $f(2) > 0$, hence by intermediate value theorem the root of equation $f(x) = 0$ lies in interval $(1.5, 2) = (a, b)$

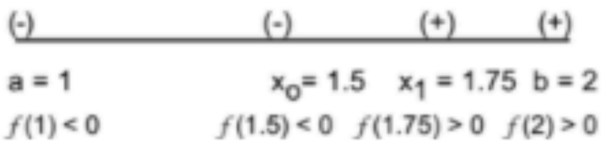
Let x_1 be the second approximation to the root.

By bisection formula

$$x_1 = (a + b)/2 = (1.5 + 2)/2 = 3.5/2 = 1.75$$

- **Second Iteration (Third approximation) :**

$$\text{Now, } f(1.75) = (1.75)^2 - 3$$
$$= 3.0625 - 3$$
$$= 0.0625 > 0 \text{ (positive)}$$



Now, $f(1.5) < 0$ and $f(1.75) > 0$ and , hence by intermediate value theorem the root of equation $f(x) = 0$ lies in interval $(1.5, 1.75) = (a, b)$

Let, x_2 be the third approximation to the root.

By bisection formula,

$$x_2 = (a + b)/2 = (1.5 + 1.75)/2 = 3.25/2 = 1.625$$

After two iterations by the Bisection method $\sqrt{3} = 1.625$

Q2) Find roots of $f(x) = x^2 - \sin(x) - 0.5$, for $[a,b] = [0,2]$, $\epsilon = 10^{-3}$; note $f(0)f(2) < 0$.

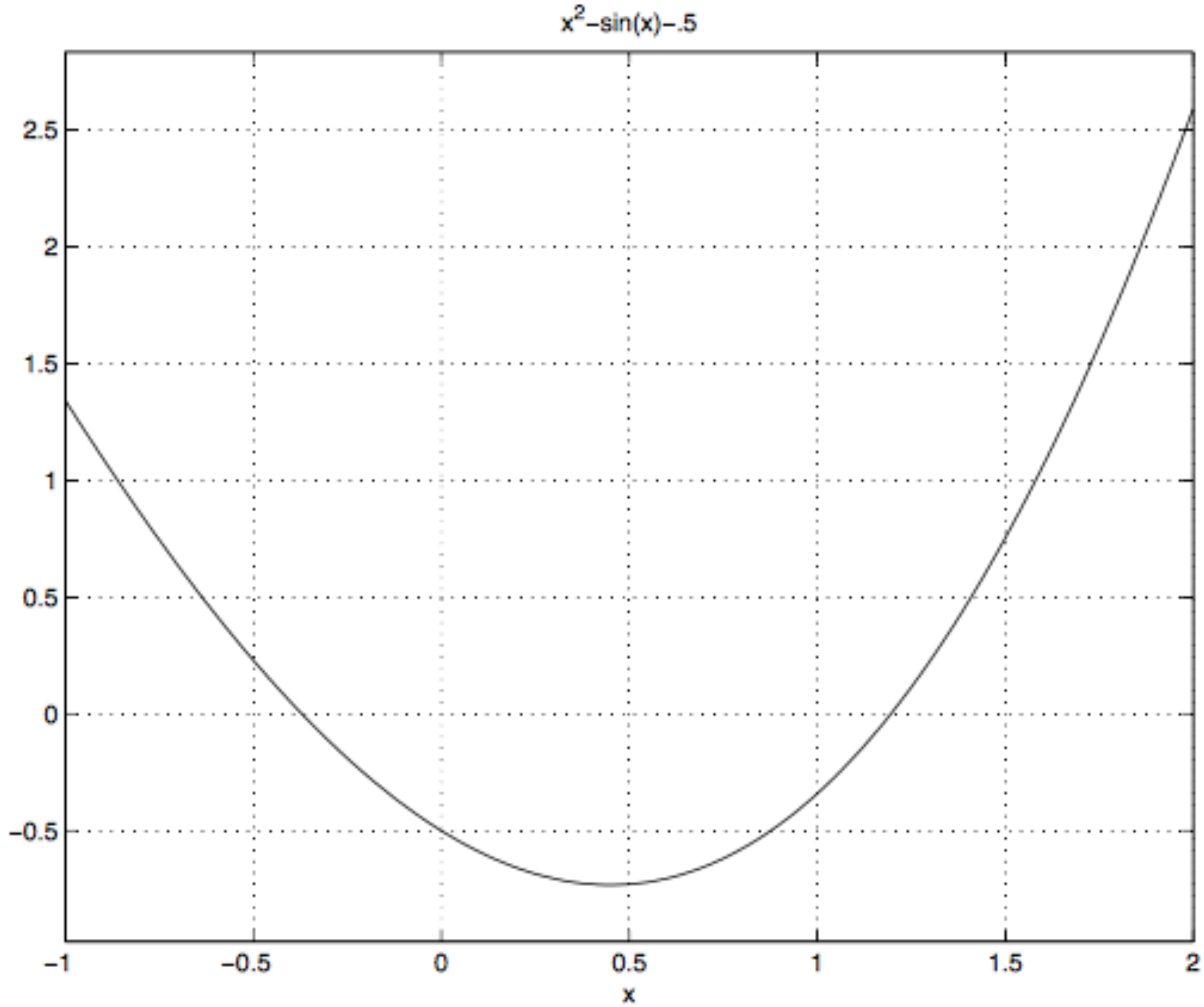
i	a	r	b	(b-a)/2
1	0	1	2	1
2	1	1.5	2	0.5
3	1	1.25	1.5	0.25
4	1	1.125	1.25	0.125
5	1.125	1.1875	1.25	0.0625
6	1.1875	1.2188	1.25	0.03125
7	1.1875	1.2031	1.2188	0.015625
8	1.1875	1.1953	1.2031	0.0078125
9	1.1953	1.1992	1.2031	0.0039062
10	1.1953	1.1973	1.1992	0.0019531
11	1.1953	1.1963	1.1973	0.00097656

$f(1.1963) = -0.0015836$;

$[a,b] = [-1,0]$?

1	-1	-0.5	0	0.5
2	-0.5	-0.25	0	0.25
3	-0.5	-0.375	-0.25	0.125
4	-0.375	-0.3125	-0.25	0.0625
5	-0.375	-0.34375	-0.3125	0.03125
6	-0.375	-0.35938	-0.34375	0.015625
7	-0.375	-0.36719	-0.35938	0.0078125
8	-0.375	-0.37109	-0.36719	0.0039062
9	-0.37109	-0.36914	-0.36719	0.0019531
10	-0.37109	-0.37012	-0.36914	0.00097656

$f(-.37012) = -0.0012839$.



x^2–sin(x)–.5
2.5
2
1.5
1
0.5
0
–0.5
–1
–0.5
0
0.5
1
1.5
2
x