How to simulate a Poisson distribution using the *crude* Monte Carlo method

We will illustrate this technique with the help of an example.

Question:

A street artist performs a magic show in the main street of Mingletown for three hours. He hopes to earn enough for a bottle of beer which costs INR 350. Throughout the three hours, people give him coins at random. So we will assume that the coins arrive in his bag according to a Poisson distribution. The amount of money each person gives is random with distribution

$$\begin{aligned} P(\text{INR } 5) &= 0.4 \\ P(\text{INR } 10) &= 0.4 \\ P(\text{INR } 20) &= 0.2 \end{aligned} \tag{1}$$

On average, 5 people per hour give the street artist money. This implies that the Poisson process has intensity $\lambda = 5$. What is the probability the artist accumulates enough money to get his beer? That is, what is $\hat{l} = P(X_3 \geq 350)$ where X_i is the money accumulated after $i = 1,2,3$ hours?

Solution:

We can estimate this easily using the Monte Carlo simulation which makes use of the *strong law of large numbers*. The estimate $\hat{l}$ is $\frac{1}{N}\sum_{j=1}^{N} Z_j$ where Z_j is a Bernoulli random variable with output 0 or 1 depending on whether the i^{th} iteration of the Monte Carlo method resulted in the artist bagging enough money for the beer. The *strong law of large numbers* states that $\hat{l} \rightarrow E(Z_j)$ as $n \rightarrow \infty$ almost surely. Succinctly, $\hat{l} = P(X_3 \geq 350) = E\left(\mathbb{I}\left(X_3 \geq 350\right)\right)$ where we have used the *indicator random variable.*

Pseudocode

```
% Monte Carlo Simulation of Compound Poisson process

t = 3;

lambda = 5;

N = 10^6; % Number of Monte Carlo Steps

beer_price = 350;

Define an array called beer of size N x 1
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```
run a i-loop N times
    n = sample a random number from the Poisson distribution % use poissrnd
    if n > 0
        Define an array called coins of size n x 1
        run a j-loop n times
            U = sample a random number between 0 and 1 % use rand
            collect the amount in coins prescribed by the p.m.f. in eq. (1)
        end
    end
beer(i) = 0 or 1 depending on whether sum(coins) >= beer_price
end
l_hat = mean(beer)% l_hat = P(X3 >= beer_price)
relErr_hat = std(beer) / (l_hat * sqrt(N)) % relative error of l_hat by crude
Monte Carlo simulation
```