Basics of probability & Statistics

Defⁿ (Probability) :- Measure of the likelihood that an event will occur

(Statistics) :- Branch of mathematics dealing w/ the collection, organization, analysis, interpretation & presentation of data.

This branch of math. relies on the principles of probability.

* Remember! Statistics **tells** you what is most likely going to happen but <u>not</u> <u>why</u> something happens!

Defⁿ (Probability space).

$$(\Omega, \mathcal{F}, \mathbb{P}).$$

Ω → Sample space (the set of all possible outcomes)

$\mathcal{F}$ → σ-algebra (collection of all events, not necessarily elementary, that we would like to consider).

$\mathbb{P}$ → Probability measure b/n $[0, 1]$. eg. $P(H) = P(T) = 0.5$

eg: tossing of a fair coin → H, T so $\Omega = \{H, T\}$.

$\mathcal{F}$ or σ-algebra (i.e. collection of all events)

No toss → $\{\ \}$ or ϕ (empty)

Toss it twice → $\{(H,H); (H,T); (T,H); (T,T)\}$

Toss it thrice → $\{(H,H,H); (H,T,T); (H,H,T); \ldots; (T,T,T)\}$

Axioms of probability

(i) If E is an event; then $P(E) \geq 0$ (or $P(E) \in [0,1]$).

(ii) $P(\Omega) = 1$ (something is bound to happen w/ certainty).

(iii) $E_1, E_2, E_3, \ldots$ Countable, disjoint events that partition ~~the~~ Ω

then $P\left(\bigcup_{i=1}^{\infty} E_i\right) = P(E_1 \cup E_2 \cup E_3 \cup \ldots) = \sum_{i=1}^{n} P(E_i)$

$= P(E_1) + P(E_2) + \ldots$

Some other useful results.

1) A, B are from Ω

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

11) If A & B are independent events $\Rightarrow P(A \cap B) = P(A)P(B)$

iii) $P(A') = 1 - P(A)$; A' means (A-complement).

eg:

Q) Roll 2 dice.

$$\Omega = \{1,2,3,4,5,6\} \times \{1,2,3,4,5,6\}$$
$$= \{(1,1);(1,2);(1,3); \ldots \ldots (6,4);(6,5);(6,6)\}$$

P(obtaining sum of outcomes ≥ 10) $= P(A)$

$= \frac{6}{36} = \frac{1}{6}$ b/c $\{(4,6);(5,5);(5,6)$ $(6,4);(6,5);(6,6)\}$ are A & total no. of outcomes $= 36$.

Def^n (Random variable, RV).

X is a variable whose possible values are outcomes of a random phenomena.

$(X \in \mathcal{F})$.

eg. Indicator RV

$$\mathbb{1}_A(\omega) = I\{\omega \in A\} = \begin{cases} 1 ; & \omega \in A \\ 0 ; & \omega \notin A . \end{cases}$$

Distribution of a RV

→ C.D.F.

→ P.M.F. (or pdf)

discrete RV; $f(x_i)$

continuous RV; $f_X(x)$

CDF (Cumulative D^n. f^n)

$$F(x) = P(X \le x)$$

i) F is non-decreasing.

ii) $F(x) \longrightarrow 1$ as $x \to \infty$

$\longrightarrow 0$ as $x \to -\infty$

iii) $F(x)$ is right continuous

$$P(X = x_i) = f(x_i)$$

$$F_X(x) = P(X \le x) = \sum_{x_i \le x} P(X = x_i)$$

$$F_X(x) = \int_{-\infty}^{x} f_X(x)\,dx$$

Conditional probability.

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

If A & B are independent events

$P(A \cap B) = P(A)\,P(B) \Rightarrow P(A|B) = P(A)$

eg: A = event that "HT" appears on 2 successive tosses of a fair coin

B = event that 1st toss = H

1st method :-

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{2} \times \frac{1}{2}}{\frac{1}{2}} = \frac{1}{2}$$

What is $A \cap B$?

HT b/c it automatically guarantees B.

But B does not guarantee A !

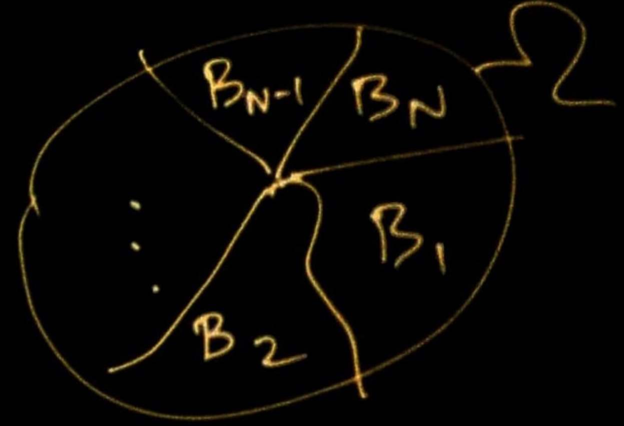
2nd method

$P(A|B) = P(2^{nd}$ toss is $T)$

$= \frac{1}{2}$ ✓

Law of total probability.

$$P(A) = \sum_{i=1}^{N} P(A|B_i)P(B_i)$$ where



Baye's thm

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B \cap A)}{P(B)} = \frac{P(B|A)P(A)}{P(B)}$$

b/c $P(B|A) = \dfrac{P(B \cap A)}{P(A)}$