Time Series Models: introductory concepts

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Time series data

Consider a time series data: $\{y_t\}_{t\geq 0} = \{y_0, y_1, y_2, ..., y_T, ...\}$

eg. amount of rainfall in a year, here t can represent the month, and y_t can represent average monthly rainfall;

eg. Gaussian white noise: $y_t = \varepsilon_t$ where $\varepsilon_t \sim \mathrm{N}(0, \sigma^2)$ are independent random variables.

Properties of white noise

1. $E[\varepsilon_t] = 0$,
2. $E[\varepsilon_t^2] = \sigma^2$, and
3. $E[\varepsilon_t \varepsilon_\tau] = 0 \quad \forall t \neq \tau$.

Realizations

First realization: $\{y_l^{(1)}\}_{l\geq 0}$
Second realization: $\{y_l^{(2)}\}_{l\geq 0}$
Third realization: $\{y_l^{(3)}\}_{l\geq 0}$
.
. $\xrightarrow{\text{sampled at t}} \{y_t^{(1)}, y_t^{(2)}, y_t^{(3)}, ..., y_t^{(I)}\}$
.
I^{th} realization: $\{y_l^{(1)}\}_{l\geq 0}$

Thus we construct a sample of I realizations of random variable Y_t

Covariance and Auto-covariance

Variance: $\gamma_{0t} := E(Y_t - \mu_t)^2$

Auto-covariance: $\gamma_{jt} := E(Y_t - \mu_t)(Y_{t-j} - \mu_{t-j})$

This is similar to $Cov(X, Y) = E(X - \mu_X)(Y - \mu_Y)$.

Stationarity

We will restrict our discussion to weak stationarity.

1. $E[Y_t] = \mu$ (independent of time),
2. $E(Y_t - \mu_t)(Y_{t-j} - \mu_{t-j}) = \gamma_j$ (independent of time), and
3. symmetry: $\gamma_j = \gamma_{-j}$ (obvious from definition).

Ergodicity

Definition: A time series process is ergodic when time averages of the random entries of the sample can be replaced by their ensemble averages.
i.e. $\bar{y} \xrightarrow{p} E[Y_t]$ where $\bar{y} := \frac{1}{T}\sum_{t=1}^{T} y_t^{(1)}$.

The above holds when $\gamma_j \to 0$ sufficiently fast as $j \to \infty$ **iff** $\sum_{j\geq 0} |\gamma_j| < \infty$.