Q) Why do we study $A\bar{x} = \bar{b}$?

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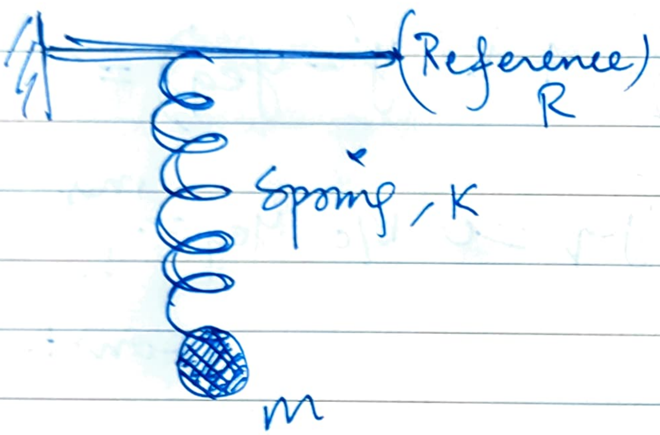
Lec-1: Simple Spring-mass system



$F_{net} = mg - Ky$; y is displ. of mass

At eq^m, $F_{net} = 0 \Rightarrow y_{eq} = \frac{mg}{K}$ —— ①

Now, pull the mass down further by y so that total elongation of spring is $(y + y_{eq})$

$$F_{net} = mg - K(y + y_{eq})$$

$$m\frac{d^2y}{dt^2} = mg - Ky - mg \quad \text{(b/c of eq(1))}$$

$$m\frac{d^2y}{dt^2} + Ky = 0 \longrightarrow ②$$

this is the model for displacement of mass from eq^n pos^n.

If we want to write an eq^n for displacement of mass w.r.t. the reference rod, R; change variables

$$y' = y + y_{eq} \longrightarrow ③$$

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or $y = y' - y_{eq}$

from eq(2): $m\frac{d^2y'}{dt^2} + k(y' - y_{eq}) = 0$

b/c $\frac{d^2}{dt}y_{eq} = 0$ b/c $y_{eq} = \frac{mg}{k}$ $= \text{const.}$

$$m\frac{d^2y'}{dt^2} + ky' = ky_{eq} = k\frac{mg}{k}$$

or $m\frac{d^2y'}{dt^2} + ky' = mg$

or $\frac{d^2y'}{dt^2} + \omega^2 y' = g$ ——— (4)

where $\omega^2 = \frac{k}{m}$

Now that we have derived the model; I can just relabel y' as x; so,

$$\frac{d^2x}{dt^2} + \omega^2 x = g \quad \text{——— (4)}$$

Let's now write eq (4) in matrix-vector form:

$$\bar{\eta} = \begin{pmatrix} x \\ \frac{dx}{dt} \end{pmatrix} = \begin{pmatrix} x \\ v \end{pmatrix}$$

So eq (4) $\equiv$ the following:

$$\frac{d^{\cancel{2}}\bar{\eta}}{dt^{\cancel{2}}} = \begin{pmatrix} 0 & 1 \\ -\omega^2 & 0 \end{pmatrix} \bar{\eta} + \begin{pmatrix} 0 \\ g \end{pmatrix} \quad \text{— (5)}$$

$$\frac{d^{\cancel{2}}\bar{\eta}}{dt^{\cancel{2}}} = A\bar{\eta} + \bar{b}_g$$

For steady state response of the spring-mass system, demand $\frac{d\bar{\eta}}{dt} = 0$

i.e. $A\bar{\eta} = \bar{b}$ — (6)

Where $\bar{b} = -\vec{b}_g$

This is how the linear-sys. $A\bar{\eta} = \bar{b}$ emerges in a simple-spring mass system.

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* Soln of w/ oscillatory modes exist if $\mathbb{A}^{-1}$ exists.

* $\mathbb{A}$ is invertible if

$$\det(\mathbb{A}) = \omega^2 \neq 0 .$$

(True whenever $k \neq 0$ or $m \neq \infty$).

* eigenvalue (λ) of $\mathbb{A} = i\omega$.

$\Rightarrow$ if $\omega = 0 \Rightarrow \lambda(\mathbb{A}) = 0$

$\Rightarrow$ $\mathbb{A}$ is NOT invertible

$\Rightarrow$ No oscillatory solns!

* Check out the python simulations & animations available on the course website

- In the next lecture, we will study a linear chain of 10 - spring mass system!