Misadventures of Squeaky the squirrel

Objective of the experiment: To illustrate elementary concepts in modeling and simulation of a random process (random walk).

Learning concepts: random number generation, random walks, stochastic simulations.

Theoretical Concepts

Random walk: A random walk is a stochastic process where the state of the system at any given instant moves to one of two possible states with equal probability. The state of the system undergoes a slow gradual drift from the initial state and is phenomenologically related to *Brownian motion* which has diverse applications in the natural sciences, economics and engineering. The term random walk was first introduced by Karl Pearson in 1905.

We will investigate the behavior of a random walk by considering the fate of a squirrel, hopping to the left or right with equal probability at every instant of time, on a one-dimensional island that abruptly drops to pits (cliffs) on either ends.

Consider a squirrel on a lonely one-dimensional island. At every instant of time, the squirrel makes a jump to the left or to the right with equal probability. His decision on the direction of his jump at any given instant is independent of all his previous such decisions; after all, he is a *happy go merry* squirrel ☺. Let us say the one-dimensional island can be explored on an n-point discrete lattice and that the squirrel commences his exploration at the position m units from the left (indexed 0 in our convention). This is a classic random walk model. Our objective is to investigate the following using the algorithm provided in the next section:

1. what is the probability that the squirrel eventually drifts to either end of the island and perishes?
2. what is his life expectancy in terms of expected number of steps to death?

Questions

Use the results of your code to answer the following questions.

1. Consider an island defined by a one-dimensional grid of length 100 units and the initial position of the squirrel is 40 units from the left (tagged 0), create a simulation of the squirrel hooping on the island over time.
2. Does the squirrel eventually fall of and die or does he just bounce on and off on the island in a never ending fashion? Play your simulation and justify your answer.
3. Does your answer above depend on the size of the island or the initial position of the squirrel? Repeat your experiment with different grid size and initial position to justify your answer.
4. What is the life expectancy of the squirrel? Does your answer tally with the theoretically predicted expected number of steps or is there a discrepancy? Explain why?

Software Implementation

Random walk algorithm:

Pseudocode of the random walk algorithm:

```
INPUT: grid_length, start_pos.

initialise curr_pos = start_pos;
initialise num_hops = 1;
while (curr_pos > 0 && curr_pos < grid_length)
   toss = rand(1);
   if (toss < 0.5)
      curr_pos = curr_pos - 1;
   elseif (toss >= 0.5)
      curr_pos = curr_pos + 1;
   end
   plot curr_pos and record graphic frame;
   num_hops = num_hops + 1;
end

OUTPUT: num_hops, play recorded animation.
```

Some useful matlab commands: `rand, stem, getframe, movie.`

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