Spring 2023

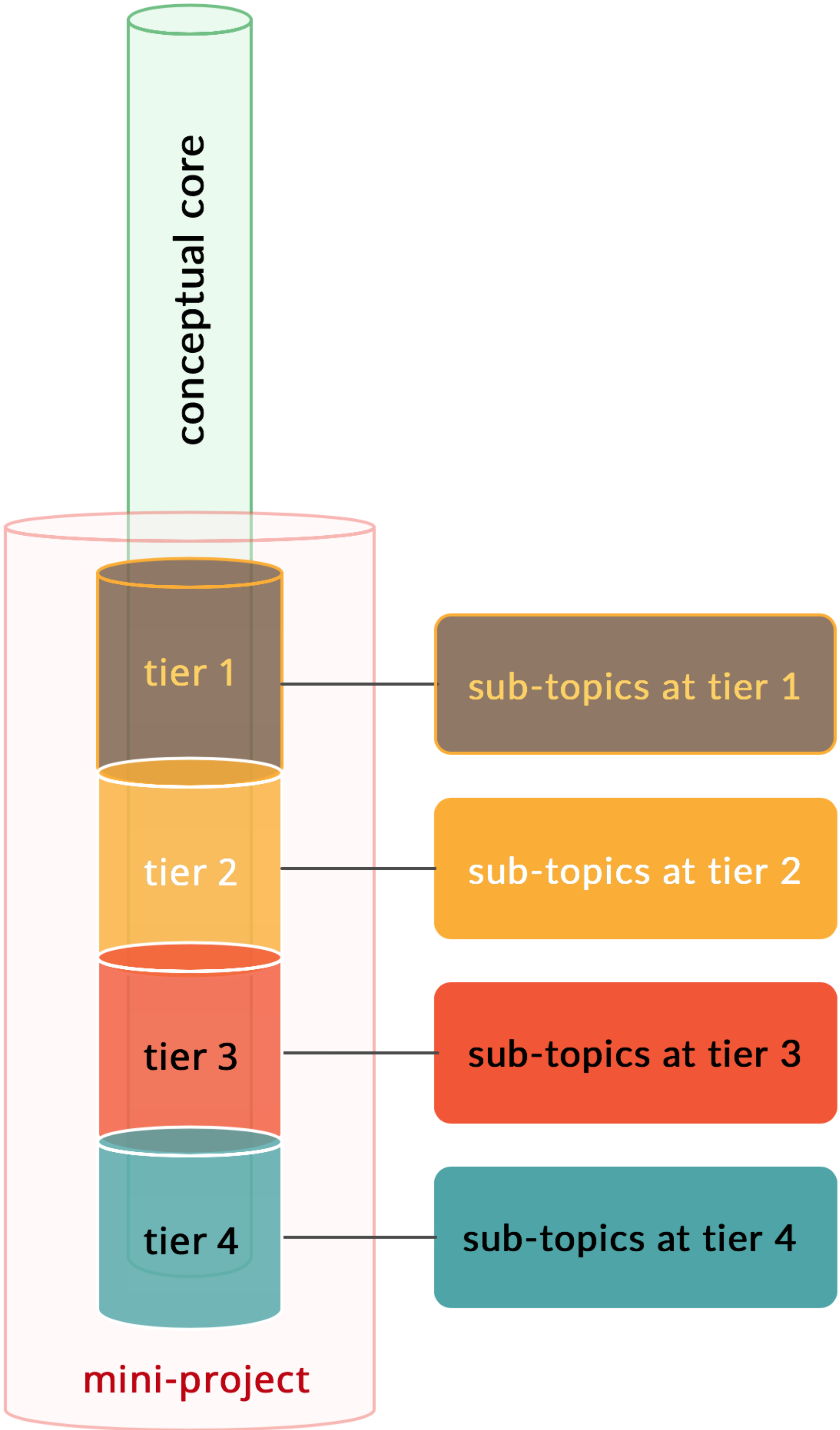
COURSE BROCHURE

Mathematics of Uncertainty

FM 133

Credits = 3 (Lecture: 2, Practice cum laboratory: 1)

Course designed by: ***Amrik Sen***



Learning unit

Contents

Chapter 1

Syllabus

1.1 Course preamble

This course will cover fundamental aspects of probability and statistics from the stand point of basic theoretical knowledge and practical applications. Students will acquire training in foundational concepts. Additionally, they will learn how to use a computer to solve diverse engineering problems by building and analysing suitable mathematical models. The course is divided into five modules. Each module comprises a *conceptual core* which is split across multiple sub-modules (tiers). Each tier will cover several related topics that will be discussed over weekly lectures and laboratory classes. There will be two lectures of 50 minutes each, one practice-cum-laboratory class of two hours every week, and one weekly tutorial hour. The course will require completion of six topical mini-projects spread across the semester.

1.2 Pre-requisites

- High school level familiarity with probability and statistics.
- Linear algebra at undergraduate level.
- Experience with a programming language (Matlab or Python).

1.3 Course instructors

- **Lead instructor**: Amrik Sen
- **Teaching Assistants**: Rohit Singla, Abhishek Thakur, Sushma, Viraj D'Souza

1.4 Recommended textbooks

All essential course related materials will be provided on the course website in the form of book chapters, laboratory manuals, and slides (when applicable) and should be sufficient for the learning expected from this course.

- ***Play of Chance and Purpose*** *- An invitation to Probability, Statistics, and Stochasticity using simulations and projects* by *Amrik Sen*, Cambridge University Press, (*in press*).

1.5 Reference books

Interested students may also refer to many classic texts on this subject as extra reference material.

1. ***Weighing the Odds*** *- A course in Probability and Statistics* by *David Williams*, first edition, Cambridge University Press, (2001).

2. ***Probability Theory*** *- The Logic of Science* by *E. T. Jaynes*, first edition, Cambridge University Press, (2003).

3. ***Introduction to Probability and Statistics for Engineers and Scientists*** by *Sheldon M. Ross*, sixth edition, Cambridge University Press, (2021).

4. ***Probability, Random Variables and Stochastic Processes*** by *Athanasios Papoulis and S. Unnikrishna Pillai*, fourth edition, McGraw Hill, (2017).

1.6 Assessments

- One mid-term exam (30%, proctured) + one quiz (10%, proctored) = 40% weightage
- Six mini-projects= 60% weightage (the sixth project may be substituted by a written quiz, this will be decided after mid-term).

Students will have to obtain a 'pass'grade separately for the written exam (theory) and the laboratory exam (projects) in order to pass the course.

1.7 Software

The following programming languages will be used for the laboratory experiments-cum-projects.

- Matlab
- Python

1.8 Course Learning Outcomes (CLOs)

Upon successful completion of this course, students should be able to acquire the following skills.

1. Calculate probabilities and expected outcomes of complex problems.
2. Model stochastic phenomena with finite memory as Markov chains and make predictions.
3. Model and analyse stationary solutions of queues and birth-death processes.
4. Analyse higher dimensional data using regression and principal component analyses.
5. Make decisions under uncertainty in relation to a variety of complex engineering problems.

Chapter 2

Course content and plan

2.1 Topics

Module 1

Conceptual core: Thinking in Probability

(i) **Tier 1:** Definition of probability space and measure, axioms of probability, random variable, conditional probability and the law of total probability.

(ii) **Tier 2:** Bayes' theorem, expected value and variance of a random variable, law of total expectation, law of iterated expectation.

Mini-project-1: Tipsy in a circus - circular random walk.

Module 2

Conceptual core: Probability Distributions

(i) **Tier 1:** Meaning of integral with respect to a probability distribution function, definition of cumulative distribution function (cdf), probability mass function (pmf), and probability density function (pdf), different types of discrete and continuous probability distribution functions, compound probability distribution, joint and marginal distribution functions.

(ii) **Tier 2:** Moment generating functions and their applications.

(iii) **Tier 3:** Asymptotic results prescribed by the law of large numbers and the central limit theorem.

Mini-project-2: Predicting insurance claim aggregates during a policy period.

Module 3

Conceptual core: Discrete Time Markov Chains

(i) **Tier 1:** Definition of Markov chains, transition probabilities, Chapman-Kolmogorov equations, distribution of states.

(ii) **Tier 2:** Recurring events in Markov chains: hitting probability, return times and absorption times, mean number of returns to a state.

(iii) **Tier 3:** Classification of Markov states and their properties.

Mini-project-3: **Automatic prediction of control laws of an aircraft using the Viterbi algorithm.**

Module 4

Conceptual core: Continuous Time Markov Chains & Queues

(i) **Tier 1:** Basics of Continuous Time Markov Chains.

(ii) **Tier 2:** Generator matrix, stationarity, detailed balance.

(iii) **Tier 3:** Birth and death processes, queues, Little's law, Pollaczek-Khinchin law.

Mini-project-4: **Queues and crowd management.**

Module 5

Conceptual core: Statistics for Complex Problems: analysis of higher dimensional data

(i) **Tier 1:** Least squares regression.

(ii) **Tier 2:** Basics of multi-variate statistics, stationarity and ergodicity, covariance matrix.

(iii) **Tier 3:** Principal Component Analysis.

Mini-project-5: **Minimalism of a butterfly.**

2.2 Weekly lecture, laboratory and project plan

A detailed week-wise lecture plan is provided below. The laboratory classes will require access to a computer/laptop and the Matlab software. You are encouraged to collaborate with your peers in small groups of 4-5 students in the laboratory. Assessments of mini-projects completed by the students at the end of every module will be done on an individual basis (no group assessments). The mini-projects are designed in such a way that the students should be able to complete the mini-projects during the allotted laboratory hours of a given module. Assessments of mini-projects will comprise three components:

1. Assessment of programming ability wherever applicable. This will include both program syntax and program logic development.

2. Assessment of the analysis of the problem and presentation of the solutions. Solutions to questions must be provided in a concise report format using mathematical editing functionalities whenever required.

3. Self and/or peer assessment by students.

The tables below present the topical layout of the lectures and laboratory for each module.

Note:
L2.1 refers to 1st lecture of 2nd week.
P2.1 refers to 1st laboratory class of 2nd week.

Module 1: *Thinking in Probability*

Week	**Lecture topics**	**Laboratory topics**
L1.1 L1.2 P1.1	Introduction to the course, discussion of syllabus and course policies, why should one study probability and statistics? Introduction to mini-project 1. Deterministic vs probabilistic outcomes, definitions of probability space and measure, axioms.	**Instruction**: Random variable (r.v.), conditional probability, law of total probability. **Matlab**: Launch mini-project 1. Getting started: Tipsy in a circus (random walk on a circle).
L2.1 L2.2 P2.1	Bayes' theorem and examples, further discussion on mini-project 1. Brief discussion on central tendency (mean, median, mode), expected value of a random variable, variance of a random value, examples.	**Instruction**: Law of total expectation. **Matlab**: Simulations. Expected life expectancy of Tipsy.
L3.1 L3.2 P3.1	More examples on expectation and the law of iterated expectation. Launch worksheet for module 1 and discussions on a few problems.	**Tutorial**: discussion on worksheet 1 (I-Mgr/TAs). **Matlab**: Mini-project 1 grand finale.
Mini-project 1	***Random walk on a circle.***	*** Mini-project 1: Submission due by the end of laboratory class P3.1.**

Module 2: *Probability Distributions*

Week	Lecture topics	Laboratory topics
L4.1	Introduction to distributions and their utility. Geometrical interpretation of integral with respect to a distribution function. Introduction to mini-project 2.	
L4.2	Different types of discrete and continuous probability distributions and their applications with numerical examples. Discussion will include concepts of cumulative distribution function (pdf), probability mass function (pmf) and probability density function (pdf).	
P4.1		**Instruction**: Detailed discussion on Compound probability distributions; eg., compound Poisson distribution. **Matlab**: Launch mini-project 2. Getting started: Predicting insurance claim aggregates during a policy period.
L5.1	Joint and marginal distributions (both discrete and continuous).	
L5.2	Numerical examples on joint and marginal distributions (including conditional expectation and variance/co-variance).	
P5.1		**Instruction**: Moment generating functions and their applications. **Matlab**: Simulations of a stochastic model to compute the associated risk for the insurance firm in terms of an appropriate probability.
L6.1	Distribution of functions of r.v., transformation of r.v., examples.	
L6.2	Asymptotic results: law of large numbers, central limit theorem.	
P6.1		**Tutorial**: discussion on worksheet 2 (I-Mgr/TAs). **Matlab**: Mini-project 2 grand finale.
Mini-project 2	***Predicting insurance claim aggregates during a policy period.***	*** Mini-project 2: Submission due by the end of laboratory class P6.1.**

Module 3: *Discrete Time Markov Chains*

Week	Lecture topics	Laboratory topics
L7.1 L7.2 P7.1	Definition of Markov chains and transition (stochastic) matrix, multi-step transition probabilities, examples. Introduction to mini-project 3. Chapman-Kolmogorov equations, distribution of states, examples.	**Instruction**: Stochastic and emission matrices, introduction to the Viterbi algorithm. **Matlab**: Launch mini-project 3. Getting started: Automatic prediction of control laws of an aircraft using the Viterbi algorithm.
L8.1 L8.2 P8.1	Recurring events in Markov chains: hitting probability, hitting and absorption times, return times and mean return time to a state. Badminton example to compute mean return time to a state.	**Instruction**: Introduction to dynamic programming. **Matlab**: Simulating the Viterbi iterates, analysing the Viterbi paths and predicting the control laws.
L9.1 L9.2 P9.1	Classification of Markov states, mean number of returns to a state, periodicity. A Markov perspective to the issue of “mind over matter”.	**Tutorial**: discussion on worksheet 3 (I-Mgr/TAs). **Matlab**: Mini-project 3 grand finale.
Mini-project 3	***Automatic prediction of control laws of an aircraft using the Viterbi algorithm.***	*** Mini-project 3: Submission due by the end of laboratory class P9.1.**

Module 4: *Continuous Time Markov Chains and Queues*

Week	Lecture topics	Laboratory topics
L10.1 L10.2 P10.1	Poisson arrival process. Basics of Continuous Time Markov Chains - Markov property, time homogeneity, holding time, Chapman Kolmogorov equations	**Instruction**: Initiation to queues **Matlab**: Launch mini-project 4. Getting started: Queues and crowd management
L11.1 L11.2 P11.1	Kolmogorov equations and the generator matrix, stationarity, and detailed balance Applications of detailed balance: Birth and death process	**Instruction**: Simulink M/M/n queues **Matlab**: Using Simulink to model queues
L12.1 L12.2 P12.1	Applications of detailed balance: Queues (Erlang T-models) Little's law, Sojourn time, M/G/n queues, Pollaczek-Khinchin law	**Tutorial**: discussion on worksheet 4 (I-Mgr/TAs). **Matlab**: Mini-project 4 grand finale.
Mini-project 4	***Queues and crowd management***	*** Mini-project 4: Submission due by the end of laboratory class P12.1.**

Module 5: *Statistics for Complex Problems*

Week	Module 5.1: **Analysis of higher-dimensional data**	
L13.1	Least squares regression for 1D data.	
L13.2	Least squares regression for multi-dimensional data.	
P13.1		**Instruction**: Linear regression (visualisation) **Python/Matlab**: Launch mini-project 5. Getting started: Data Analysis
L14.1	Introduction to multi-variate statistics, stationarity, ergodicity, covariance matrix.	
L14.2	Basics of Principal component analysis (PCA).	
P14.1		**Instruction**: PCA vs ICA **Python/Matlab**: Grand finale: Minimalism of a butterfly
Mini-project 5	**Minimalism of a butterfly**	**Submission of Project 5 by the end of lab class P14.1.**